THE THEORY OF THE FIRM UNDER INPUT PRICE UNCERTAINTY

By

RICHARD GEORGE FLOOD

A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1978

© Copyright 1978 by Richard George Flood

ACKNOWLEDGMENTS

I would like to extend my sincere appreciation and gratitude to Dr. Ira Horowitz, the chairman of my dissertation committee. Not only was his assistance and guidance instrumental in the completion of this research, but also his advice and encouragement were responsible for my entering the graduate program at the University of Florida.

Dr. Roger Blair, member of the dissertation committee, provided considerable assistance and support and made many helpful suggestions, for which I am very grateful.

Many other members of the faculty and staff of the University of Florida and fellow graduate students contributed, directly or indirectly, to my graduate work. To all of them, I would like to express my thanks.

Most importantly, I acknowledge the moral support and encouragement of my family. My parents and my wife, Lynn, provided love, confidence, and patience that helped make the completion of my studies a reality.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	iii
ABSTRACT	vi
CHAPTER	
I. INTRODUCTION	1
Purpose	1
Random Costs	2
Methodology	3
Organization	5
Objectives	5
II. LITERATURE SURVEY	7
Introduction	7
Uncertainty and the Theory of the Firm	7
Uncertainty and the Cournot Model	11
Input Price Uncertainty and the Theory of the Firm	13
Summary	16
III. THE COMPETITIVE FIRM UNDER COST UNCERTAINTY	17
Introduction	17
The Firm under Certainty	17
Rate of Technical Substitutition under Uncertainty	19

Page

Input Demand Functions and Risk Attitudes 22

Comparative Statics--The Uncertainty Case 28

Summary 37

IV. LABOR NEGOTIATIONS UNDER COST UNCERTAINTY 39

Introduction 39

Labor Contract Model 39

Increased Risk and Contracts 45

Fixed Cost of Capital 54

Summary 55

V. THE DUOPOLIST UNDER COST UNCERTAINTY 57

Introduction 57

Cournot Model under Cost Uncertainty 58

Linear Demand Function 62

General Demand Function 70

Comparative Statics 77

Differing Cost Expectations 84

Summary 85

VI. CONCLUSIONS AND RECOMMENDATIONS 87

Conclusions 87

Recommendations 89

APPENDIX

A. PROOF 92

B. PROOF 93

BIBLIOGRAPHY 95

BIOGRAPHICAL SKETCH 97

Abstract of Dissertation Presented to the Graduate Council
of the University of Florida in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy

THE THEORY OF THE FIRM UNDER INPUT
PRICE UNCERTAINTY

By

Richard George Flood

August 1978

Chairman: Ira Horowitz
Major Department: Management

The purpose of this research is to examine the theory of the firm under cost uncertainty. Empirical evidence suggests that input costs are random. As a result, the decisions of the firm should be analyzed in light of this uncertain environment. The traditional neo-classical model of the firm assumes a nonstochastic world. Uncertainty significantly alters the optimal decisions of the firm.

This research examines the effects of input price uncertainty on the perfect competitor, the duopolist, and labor negotiations. It assumes that the firm is an expected utility maximizer and uses a decision-theoretic approach to solve for the optimal strategies of the firm. The results of the analysis are compared and contrasted with those of the nonstochastic model.

The problem for the competitive firm is to choose the optimal quantities of capital and labor as to maximize profits. The cost of capital is assumed to be a random variable for which a probability

density function has been assessed. The decisions of the firm are analyzed in light of its preference for risk. It is shown that the firm may not operate at its cost minimizing capital-labor combination and that under some circumstances the entrepreneur may prefer a risky situation to a riskless one.

The assumption that the wage rate is nonstochastic is based on the observation that contracts exist between labor and the firm which fix wage rates for the duration of the agreement. The risk attitudes of the bargaining parties are analyzed as to their effects on these contracts. It is shown that the ability to assess the probability distribution of the wage rate and the utility function of the opposing party is essential for the agreement to an acceptable wage.

The behavior of the duopolist is modeled in a stochastic environment. The model is shown to result in a stable equilibrium although the industry output declines from that of a world of cost certainty. Suggestions are made for extending this research to other market structures and for incorporating other forms of uncertainty into the model.

CHAPTER I
INTRODUCTION

Purpose

The neoclassical theory of the firm assumes that production functions, demand curves, and factor supply curves are known with certainty; or, even if a degree of uncertainty exists about these functions, the entrepreneur behaves as if they are known with certainty. A particular functional form may be assumed, or the firm may estimate these forms and use the estimates as dogmatic input to its decision process. This certainty assumption leads to an unambiguous description of the optimal behavior of the firm. While almost every phase of economic behavior is affected by uncertainty, deterministic economic theory does not provide an adequate description of individual firm behavior in a stochastic environment. Recent developments in decision-making processes under uncertainty have made it possible to consider uncertainty in a formal sense.

The effects of demand and production function uncertainty have been thoroughly explored. The results indicate that optimal firm behavior is significantly modified when uncertainty is considered. The effects of stochastic input costs have, however, remained relatively unexamined. The purpose of this research is to investigate uncertain input prices as they affect the competitive firm, the duopolist, and managment-labor wage negotiations.

Random Costs

The firm, as a purchaser of inputs, faces constantly changing factor costs. Inflation is one factor that causes costs to rise, but even with the effects of inflation removed, input costs still change in what appears to be a random manner. There are a number of reasons for this phenomenon, the principal one of which is that the factor supply functions undergo random shifts. A freeze in Brazil that reduces the supply of coffee beans increases the raw material costs of the coffee producers; a one shot-decision by the government of the Soviet Union causes a shortage in the United States and, therefore, the cost of wheat rises; and the oil supply of the world changes from year-to-year as do the costs.

A second explanation for random costs is technology. Technological progress in the electronics industry, for example, has had a profound effect on costs. The development of the transistor has reduced the cost of producing radios, televisions, computers, and other electronic instruments. More effective fertilizers and pest control chemicals reduce agricultural costs.

Third, competition or the lack of competition affects costs. If there are many producers of the input, the cost may remain relatively stable, whereas, if a monopoly or a collusive oligopoly produces the input, then they may have a monopolistic control over the price of the input. The oil producing nations in the Mideast, in collusion, have frequently, and in a nonpredictable manner, changed the price of that resource.

Finally, governmental controls may cause randomness in costs. The prerogative of the government to institute price and wage controls makes costs uncertain in the long term. Corporate tax rates, tariffs, and price supports, all of which are controlled by the government, influence factor prices.

Labor costs are also constantly changing. Although the overwhelming tendency is for wages to increase over time, the existence of wage contracts does, in the short term, stabilize labor costs. Thus, the firm may face some costs that are random and others that are not. This research examines the firm in this environment.

Methodology

The entrepreneur's problem is to maximize the expected utility of profits. It is assumed that he has assessed a utility function that reflects his attitude toward risk. The methodology employed in this research is decision-theoretic. The analysis used was first suggested by John von Neumann and Oskar Morgenstern (1944). The acceptance of a set of behavioral axioms is the essential assumption for the model. This set of axioms defines rational behavior in a decision-making situation.

The two useful measurements of risk aversion were developed by Pratt (1964) and Arrow (1971). The measurements indicate the extent to which an individual wishes to avoid or accept risk. The first, which is called absolute risk aversion and is defined

$$R_A = \frac{-U''(\$)}{U'(\$)},$$

measures an individual's propensity to accept risk as a function of his wealth level. The second, which is called relative risk aversion and is defined

$$R_R = \frac{-\$\cdot U''(\$)}{U'(\$)},$$

measures the proportion of wealth that an individual is willing to invest in a risky asset as a function of his wealth level. Arrow (1971) proposed that a utility function should display decreasing absolute risk aversion.

As previously stated, the entrepreneur's presumed objective is to maximize the expected utility of profits. Profit is defined

$$\pi = P\cdot Q - \tilde{r}\cdot K - w\cdot L$$

where P is price, Q is quantity, K is the amount of capital utilized, L is the amount of labor employed, and $\tilde{r}$ and w are the costs of capital and labor, respectively. The cost of capital is a random variable that is described by a probability density function $f(\tilde{r})$. Defining a utility function in the von Neumann-Morgenstern (1944) sense, the general problem in this research is to determine factor input levels that maximize $E[U(\pi)]$.

The model and approach assume that one can define a unique utility function and a profit function that satisfy the first-order

and second-order conditions for solving a multivariable optimization problem. Attempts to empirically derive such functions have been less than successful. But if one is willing to accept, at least in theory, these assumptions, then the von Neumann-Morgenstern (1944) decision-theoretic method is appropriate for the analysis of the entrepreneur's factor-choice problem.

Organization

Chapter II gives a general review of the literature on the stochastic theory of the firm and a more detailed examination of the specific research most closely related to this study. Chapter III examines the traditional theory of the firm and introduces cost uncertainty. Contrasts and comparisons are made between optimal behavior in a world of certainty and in an uncertain environment.

Chapter IV investigates the effects of an uncertain wage rate on labor negotiations. The effect on both the firm and labor are analyzed.

Chapter V discusses the duopoly model under cost uncertainty. Again, comparisons are made with the nonstochastic case.

Chapter VI concludes the research with a summary of the results and recommendations for further research. The appendixes include mathematical proofs of some of the material presented as postulates in the main text.

Objectives

The primary objective of this research is to reexamine

traditional economic theory and firm behavior when input prices are random. The secondary objective is to demonstrate the application of decision theory to economic problems.

CHAPTER II
LITERATURE SURVEY

Introduction

There has been a rather extensive investigation of the effects of uncertainty on the theory of the firm. Most of this research has dealt with demand uncertainty; principally, uncertainty with respect to price, where output is the decision variable of the firm. Although this literature has in the main ignored input price uncertainty, it has nevertheless provided the framework for the problem area under consideration in this dissertation. The purpose of the present chapter is to give an overview of representative literature in the general area of uncertainty and the theory of the firm, followed by a more detailed discussion of the literature closely related to the topic of this research. This survey is divided into three sections as follows:

1. Uncertainty and the Theory of the Firm,
2. Uncertainty and the Cournot Model, and
3. Input Price Uncertainty and the Theory of the Firm.

For other literature surveys, the reader is referred to McCall (1971) or Baron (1970).

Uncertainty and the Theory of the Firm

The first efforts to introduce uncertainty into the theory of the firm did so by considering the firm with a linear utility

function, that is, the firm that is indifferent to risk. Price was assumed to be the stochastic element. Oi (1961) assumed that the firm could instantaneously adjust its production level. The objective of the firm was to maximize expected profits. His results indicated that a random price was preferred to a certain price by the purely competitive, risk-neutral firm. Furthermore, this preference for price uncertainty can also be seen in some firms with nonlinear utility functions.

Tisdell (1963) criticized Oi's (1961) assumption of instantaneous adjustment of production. Tisdell assumed that the firm was an expected profit maximizer that must plan output in advance of observing price. The optimal output of the firm under these circumstances is to produce at the level where expected price equals marginal cost. He also showed that the firm is indifferent to price uncertainty.

Oi's (1961) assumption is equivalent to perfect price prediction, whereas Tisdell's (1963) assumption is equivalent to no predictive ability at all. Nelson (1961) examined the quantity-setter under various degrees of price prediction ability. Assuming a quadratic total cost function, Nelson developed a model that demonstrated that as the predictive ability of the firm improved, its expected profits increased. His results, at the limits of perfect prediction and no predictive ability, corresponded to Oi's and Tisdell's results. Furthermore, Nelson showed that the value of prediction to an individual firm tends to decrease as more firms in

the industry have predictive ability. Zucker (1965), assuming a constant elasticity supply function, derived a certainty price that showed the equivalence of the Oi and Tisdell results.

Penner (1967) was one of the first researchers to investigate the effects of nonlinear utility functions on the theory of the firm. Specifically, he showed that a risk-averse firm that produces in advance of demand being known will produce less than the risk-neutral firm. Also, the risk taker will produce more than the firm with a linear utility function. Penner also investigated the effects of a profits tax. His results indicated that the output of the firm with nonlinear risk preferences is reduced when a profits tax is introduced.

Hymans (1966) and Horowitz (1969) considered the price-taker and the price-quoter under uncertainty, respectively. Hymans showed that price and supply are positively related for risk-neutral firms, but may be negatively related if the firm is risk averse. He further showed that, for any risk preferences, the firm produces less under uncertainty than it would under certainty. Horowitz showed that the prices preferred by different firms in an industry will tend to differ as their risk preferences differ. Also, he showed that, with increasing marginal costs, the risk averter prefers a price such that expected marginal revenue is less than expected marginal cost and that the risk taker prefers a price such that expected marginal revenue exceeds expected marginal cost.

Baron (1970), Sandmo (1971), and Leland (1972) examined the

competitive firm under demand uncertainty. Baron's results indicate that prices are higher and output lower in an industry where risk aversion prevails. Second, risk aversion in a purely competitive environment explains the existence of expected profit even at equilibrium. Third, the competitive firm does not operate at its most efficient output level since, at equilibrium, the marginal cost of the risk averse firm is less than average total cost. Finally, he shows that fixed costs do affect the short-run output decisions of the risk-averse firm unless its utility function displays constant absolute risk aversion. Sandmo assumed the firm must make a commitment as to its production level in a world of price uncertainty. He investigated the comparative statics of the firm and showed that the firm does react to increases in fixed costs, increases in the mean of the distribution of price, and increases in the tax rate. Decreasing absolute risk aversion is a necessary and sufficient condition for decreasing output as a result of the aforementioned changes. Furthermore, he showed that the competitive firm requires positive expected profits in order to choose a positive output level. Leland considered the case where the firm has the choice of price-setting or quantity-setting. His results indicate that the conclusions of Baron and Sandmo are special cases of a more general model. The risk-averse firm is not neutral with regard to the choice between price-setting or quantity-setting. Random demand and nondecreasing marginal cost will lead to a lower price if the firm is a price-setter and lower output if the firm sets quantity.

Other forms of market structure and firm structure have been analyzed by Mills (1959), Zabel (1970), Dhrymes (1964), and Blair and Heggestad (1977). Mills, examinining the imperfectly competitive environment, showed that the risk-neutral firm equates marginal revenue with something greater than marginal cost and that if demand is uncertain and marginal cost constant the firm will charge a lower price than if it operated in a deterministic world. Blair and Heggestad, examining the multiproduct firm under uncertainty, showed that product diversification will not necessarily lead to increased output as is generally regarded in a certainty world. Output expansion in the multiproduct firm requires that some of the product prices be negatively correlated; otherwise, the optimal output for a given commodity will be less than the single product firm.

Uncertainty and the Cournot Model

The literature concerning the oligopolist under conditions of uncertainty is quite limited. Research in this area has focused on the problem of uncertainty with regard to rival output. The major contributions have been made by Horowitz (1970), Tarr (1972), Hosomatsu (1969), and Sato and Nagatani (1967).

Sato and Nagatani (1967) investigated the stability of the oligopoly model under conditions of rival output uncertainty. They showed that this model is likely to be more stable than the Cournot model. They also presented a collusion model that was shown to have either no possibility of equilibrium or a very strong possibility of

stable equilibrium. Hosomatsu (1969) assumed that both the demand function and the outputs of the rival were unknown quantities and that the firm must estimate price and use that price in an estimated demand function to arrive at the production levels of its rival. He indicated that this model is more stable as the number of oligopolists in the market increased and as the slope of the estimated demand function became steeper in comparison to the actual demand function. Furthermore, he showed that the slope is the decisive element of the demand function for stability conditions and that knowledge of this demand function may make knowledge of the output of the rival unnecessary.

Horowitz (1970) assumed that each firm assessed a probability density function to describe the output of its rivals and that the mean of the distribution in this period is the observed output of the last period. He observed that the risk-averse firm will produce less than the risk-neutral firm and the risk taker will produce a greater quantity than the firm with linear risk preferences. Furthermore, the more diffuse the assessed density functions, the greater the difference in outputs. He pointed out that the main effect of the uncertainty was to shift the duopolist's reaction curves and, as a result, the model should lead to a stable equilibrium. Tarr (1972) made similar assumptions which led to his conclusions that the market will tend to be more stable the faster marginal costs increase at equilibrium and the more insensitive the density function is to parameter changes. He also investigated specific results for the case of

linear demand and quadratic costs.

Input Price Uncertainty and the Theory of the Firm

The literature that has investigated the topic of input price uncertainty is also rather limited. There have been five major research efforts that have concentrated on this area or related areas. Batra and Ullah (1974) and Holthausen (1976) have analyzed input demand theory under price uncertainty, Rothenberg and Smith (1971) have researched the effects of resource supply uncertainty on the allocation of resources, Doeleman and Stahl (1975) have looked at the effects of inflation uncertainty on labor union wage demands, and only Blair (1974) has investigated random input prices and the theory of the firm, but with limited results. This section takes a more detailed examination of these efforts as they are most closely related to the topic of this research.

Batra and Ullah (1974) examined the expected-utility-maximizing, price-taking firm that faces an uncertain sales price. They were concerned about the input choices made by the firm, prior to observing the market price. Analyzing a two-input model, they determined, for a risk-neutral firm, that a concave production function is a necessary and sufficient condition for optimality where the first-order conditions hold. If the firm is risk averse, concavity is a sufficient but not necessary condition for optimality where the first-order conditions hold. Furthermore, for the risk taker, convexity is both necessary and sufficient to ensure that the

firm reaches an optimum where first-order conditions are met. They also showed that price uncertainty will cause the firm to use smaller amounts of the inputs and, therefore, lower their output. As the uncertainty increases, output declines further if the absolute risk aversion of the firm is decreasing with respect to profits. Finally, they indicated that a "well-behaved" production function, a function for which the second partial derivatives are negative and the cross partial derivatives are positive, is necessary to ensure that a rise in the price of an input causes the firm to use less of that input. They were unable to ascertain the effects of this price increase on the utilization of the other input.

Hartman (1975) criticized the Batra and Ullah (1974) result that the risk-averse firm utilizes smaller input quantities. He argues that the correct approach to the problem is for the firm to first choose its output level and then choose the inputs to achieve this level of output at minimum cost to the firm. His results show that under uncertainty, the firm produces less, but that this smaller output ambiguously affects input demand.

Holthausen (1976) analyzed the imperfect competitor under demand uncertainty. His results indicate that, for the price-setting imperfect competitor, the risk-averse firm will use an expected capital-labor ratio that is less than the optimal ratio under certainty and the risk taker will use a ratio that is larger. The capital-labor ratio of the risk-averse firm approaches the efficient ratio as the wealth of the firm increases provided, however, that the

firm displays decreasing absolute risk aversion.

Rothenberg and Smith (1971) examined two models. The first assumed that the supply of labor services is a random variable. A result of this model is that the more labor-intensive is the firm, the more likely it is that the introduction of uncertainty will lead to greater utilization of capital. The second model assumed that the supply of labor was known, but that the production function contained a random component. If this random parameter occurs in a labor-intensive industry, the effect is to lower the average wage bill, while in a capital-intensive industry the wage bill increases.

Doeleman and Stahl (1975) assumed wages to be a function of the rate of unemployment and the expected rate of inflation, and that the labor union seeks an improvement in real wages equal to a percentage improvement in labor productivity. Their model examined the wage demands of the labor union and the effects of the uncertain inflation rate. Their results indicated that unions will attempt to raise their wages by a percentage in excess of expected inflation and anticipated growth of labor productivity. They also suggested that wage or tax indexation may be worthwhile as anti-inflationary tools as they remove uncertainty. They qualify this result by requiring that the inflation rate be the only source of uncertainty.

Blair's (1974) research is most closely related to this topic. He assumed the firm is committed to its output prior to knowing the input costs. All input costs are assumed random variables. The demand and production functions are assumed nonstochastic.

His results indicate that the expansion path of two risk-averse firms facing identical demand, production, and input supply functions may not be the same. He also examined supply contracts and showed that risk attitudes determine the contract terms. Finally, he attempted to measure the optimal response to increased input prices but was not able to unambiguously calculate those results.

Summary

Although there is an extensive amount of literature in the general area of the stochastic theory of the firm, the research in the specific area of input price uncertainty is limited. There has been no attempt to investigate duopoly theory under input price uncertainty and, while the competitive firm in this environment has been studied, some of the results have been ambiguous or indeterminate.

This dissertation attempts to fill these voids in the literature. The Cournot model is modified to incorporate cost uncertainty and the effect of this uncertainty is analyzed with respect to output decisions and equilibrium stability. Also, by examining the competitive firm with a two-input production process where one of the input prices is random and the other fixed, the ambiguity of previous research is eliminated.

CHAPTER III
THE COMPETITIVE FIRM UNDER COST UNCERTAINTY

Introduction

The purpose of this chapter is to set out the traditional theory of the firm as a framework within which contrasts and comparisons may be made as uncertainty is introduced. The optimizing behavior of the expected value maximizer and the expected utility maximizer is examined with specific references to their attitudes toward uncertainty. It will be shown that, under some circumstances, even the risk-averse firm will prefer the uncertain situation.

The Firm under Certainty

Consider a single-product, profit-maximizing firm whose two-input production process, $Q = q(K,L)$, utilizes capital (K) and labor (L) to produce output Q. The relationship is defined such that the second partial derivatives $\partial^2 q/\partial K^2$ and $\partial^2 q/\partial L^2$ exist and are negative, so that the function q possesses the characteristic of diminishing marginal productivity. Further, q is strictly concave in K and L, or $[\partial^2 q/\partial K^2][\partial^2 q/\partial L^2]-[\partial^2 q/\partial K \partial L]^2 > 0$. The firm is a competitive price-taking firm.

The cost of capital is represented by r, and w is the wage rate. These costs are assumed constant. Thus, profit (π) can be formulated as

$$\pi = PQ - rK - wL.$$

The entrepreneur, desiring to obtain the greatest output for a given cost, maximizes this function with respect to the inputs, thus deriving the optimal capital-labor allocations.

$$\frac{\partial \pi}{\partial K} = P\frac{\partial Q}{\partial K} - r = 0.$$

$$\frac{\partial \pi}{\partial L} = P\frac{\partial Q}{\partial L} - w = 0.$$

(Q_K and Q_L will be used to represent the marginal productivities.) Thus, solving the first-order conditions,

$$\frac{Q_K}{Q_L} = \frac{r}{w}.$$

The optimal input combination equates the ratio of the marginal physical products with the ratio of their respective prices. This ratio measures the reduction in one input per unit increase in the other that is just sufficient to maintain a constant rate of output.

The input demand functions for the producer can be derived by implicitly solving for K and L in terms of P, w, and r. It is easily shown that (Henderson and Quandt, 1971):

$$\frac{\partial K}{\partial r} = \frac{Q_{LL}}{P(Q_{KK}Q_{LL} - (Q_{KL})^2)} < 0$$

and

$$\frac{\partial K}{\partial w} = \frac{-Q_{KL}}{P(Q_{KK}Q_{LL} - (Q_{KL})^2)} < 0$$

since $Q_{KL} > 0$ when the factors are complements, as is generally assumed to be the case. Thus, the input demand functions are downward sloping.

These are the fundamental concepts of the traditional theory of the firm. The next section examines these same properties under the assumption of input price uncertainty.

Rate of Technical Substitution under Uncertainty

Consider a firm for which the cost of capital is a random variable with probability density function $f(r)$, mean $\bar{r}$, and variance $\sigma_r^2 > 0$. The wage rate, w, is assumed fixed by contract. The problem of the firm is to determine the quantities of capital and labor that maximize the expected utility of profit. Assuming a utility function,

$$U = u(\pi),$$

in the von Neumann-Morgenstern sense, the problem may be formulated as

$$\underset{K,L}{\text{Maximize}}\ Z = E[U(P \cdot q(K,L) - \tilde{r}K - wL)]$$

The first-order conditions for a maximum are

$$\frac{\partial Z}{\partial L} = E[U'(\pi)\ (PQ_K - \tilde{r})] = 0 \qquad (1)$$

$$\frac{\partial Z}{\partial L} = E[U'(\pi)\ (PQ_L - w)] = 0 \qquad (2)$$

Assume the second-order conditions are met. That is,

$$\frac{\partial^2 Z}{\partial K^2} = A_1 = E[U''(\pi)\ (PQ_K - \tilde{r})^2 + U'(\pi) \cdot PQ_{KK}] < 0$$

$$\frac{\partial^2 Z}{\partial L^2} = A_2 = E[U''(\pi)(PQ_L - w)^2 \quad U'(\pi) \cdot PQ_{LL}] < 0$$

and

$$A_1 A_2 - B_1^{\ 2} = D > 0$$

where

$$B_1 = \frac{\partial^2 Z}{\partial K \partial L} = E[U''(\pi)\ (PQ_K - \tilde{r})(PQ_L - w) + U'(\pi) PQ_{KL}]$$

$U''(\pi)$ may be greater than, less than, or equal to zero denoting risk preference, risk aversion, or risk neutrality. $U'(\pi)$ must be positive.

Manipulation of (1) and (2) indicates that the optimal allocation of capital and labor must satisfy

$$\frac{Q_K}{Q_L} = \frac{\{E[\tilde{r}]\cdot E[U'(\pi)] + \mathrm{cov}(U'(\pi), \tilde{r})\}}{E[U'(\pi)]w} \tag{3}$$

This condition is in contrast to the usual optimality conditions that arise under certainty, and a comparison may be useful at this point. Under certainty, the firm would operate at Point A (see Figure 1), where the isocost line is tangent to the production isoquant. At that point, the negative of the ratio of the costs of capital and labor equals the ratio of the marginal products. Assuming for the moment that output remains unchanged, except for the risk-neutral case (where the covariance term equals zero), the firm will choose its capital and labor combination such that the slope of the isocost line equals the ratio in (3). Point B, as noted, is the optimal allocation given output is fixed. However, the introduction of uncertainty into the model may lead to a different optimal output and, consequently, a different optimal combination of capital and labor. Point B is not necessarily positioned as in Figure 1; however, as long as it is not at A, the firm is not at its cost minimizing capital-labor combination.

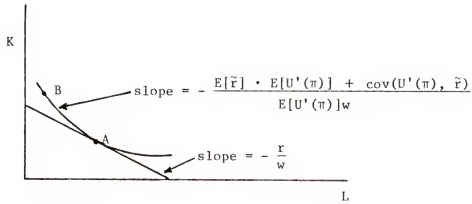

Figure 1

Input Demand Functions and Risk Attitude

Consider the risk-neutral firm that behaves as an expected value maximizer, is a price taker, and may choose its production level after observing the value of r. Assume the cost of labor is still fixed and the cost of capital is fixed at the expected value of a density function, $f(\tilde{r})$, or

$$r = \bar{r}.$$

Then, maximizing profit

$$\pi = P \cdot Q - w \cdot L - \bar{r} \cdot K$$

with respect to K and L gives the first-order conditions:

$$\frac{\partial \pi}{\partial K} = PQ_K - \bar{r} = 0$$

$$\frac{\partial \pi}{\partial L} = PQ_L - w = 0.$$

Solving implicitly for the optimal values of capital and labor, K^* and L^*, yields

$$K^* = K^*(P,\bar{r},L)$$

$$L^* = L^*(P,w,K), \text{ and}$$

$$Q^* = q(K^*,L^*).$$

The optimal level of profit, π^*, can be expressed as a function of K^*, L^*, and Q^*.

$$\pi^* = PQ^* - wL^* - \bar{r}K^*.$$

Differentiating with respect to $\bar{r}$ gives us

$$\begin{aligned}\frac{\partial \pi^*}{\partial \bar{r}} &= P\frac{\partial Q^*}{\partial K^*}\frac{\partial K^*}{\partial \bar{r}} + P\frac{\partial Q^*}{\partial L^*}\frac{\partial L^*}{\partial \bar{r}} \\ &\quad - w\frac{\partial L^*}{\partial \bar{r}} - K^* - \bar{r}\frac{\partial K^*}{\partial \bar{r}} \\ &= (P\frac{\partial Q^*}{\partial K^*} - \bar{r})\frac{\partial K^*}{\partial \bar{r}} + (P\frac{\partial Q^*}{\partial L^*} - w)\frac{\partial L^*}{\partial \bar{r}} \\ &\quad - K^*. \qquad (4)\end{aligned}$$

From the first-order conditions,

$$P\frac{\partial Q^*}{\partial K^*} - \bar{r} = 0 \text{ and}$$

$$P\frac{\partial Q^*}{\partial L^*} - w = 0.$$

Therefore,

$$\frac{\partial \pi^*}{\partial \bar{r}} = -K^*, \tag{5}$$

which is negative since $K^* > 0$. This indicates that π^* is a decreasing function of $\bar{r}$. An examination of $\frac{\partial^2 \pi^*}{\partial \bar{r}^2}$ will show that π^* is not only a decreasing function of $\bar{r}$, but also a convex function. This will lead to the result that the risk-neutral firm prefers uncertainty to certainty.

Differentiating (5) again with respect to $\bar{r}$,

$$\frac{\partial^2 \pi^*}{\partial \bar{r}^2} = -\frac{\partial K^*}{\partial \bar{r}} \tag{6}$$

Differentiating (4) again with respect to $\bar{r}$,

$$\frac{\partial^2 \pi^*}{\partial \bar{r}^2} = P[\frac{\partial K^*}{\partial \bar{r}}(Q^*_{KK}\frac{\partial K^*}{\partial \bar{r}} + Q^*_{KL}\frac{\partial L^*}{\partial \bar{r}}$$

$$+ Q^*_{KL}\frac{\partial L^*}{\partial K^*}\frac{\partial K^*}{\partial \bar{r}})] + P[\frac{\partial L^*}{\partial K^*}\frac{\partial K^*}{\partial \bar{r}}$$

$$(Q^*_{KL}\frac{\partial K^*}{\partial \bar{r}} + Q^*_{LL}\frac{\partial L^*}{\partial K^*}\frac{\partial K^*}{\partial \bar{r}})]$$

$$-2\frac{\partial K^*}{\partial \bar{r}}. \tag{7}$$

Then, equating (6) and (7) and solving for $\frac{\partial K^*}{\partial \bar{r}}$,

$$\frac{\partial K^*}{\partial \bar{r}} = \frac{1}{P(Q^*_{KK} + 2Q^*_{KL}\frac{\partial L^*}{\partial K^*} + Q^*_{LL}(\frac{\partial L^*}{\partial K^*})^2)}.$$

Since $Q_{KK} < 0$, $Q_{LL} < 0$, $Q_{KL} > 0$, and $\frac{\partial L^*}{\partial K^*} < 0$, $\frac{\partial K^*}{\partial \bar{r}} < 0$.

But, from (6) we know that

$$\frac{\partial^2 \pi^*}{\partial \bar{r}^2} = \frac{-\partial K^*}{\partial \bar{r}} > 0.$$

Thus, π^* is a convex function of $\bar{r}$. With r fixed at the expected value, the relationship between π^* and $\bar{r}$ is nonlinear. This results from the characteristics of the production function.

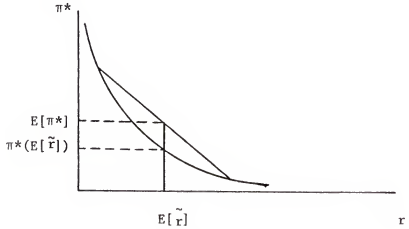

Figure 2

Holding the expected value of r constant, Figure 2 indicates that $E[\pi^*]$ increases as the variability of f(r) increases. Thus, the expected value maximizer, the risk-neutral firm, prefers the increase in variability, that is, uncertainty is preferred to certainty. This result is similar to that of Oi (1961) and also has been observed by Rothenberg and Smith (1971).

To extend this analysis to include the risk-averse firm and the risk-taking firm, assume the firm wishes to maximize utility. Once again, it will be assumed that w and r are fixed as in the previous case. Then, the problem becomes,

$$\max_{K,L} U = U(\pi).$$

The first-order conditions are

$$\frac{\partial U}{\partial K} = U'(\pi)(PQ_K - \bar{r}) = 0$$

$$\frac{\partial U}{\partial L} = U'(\pi)(PQ_L - w) = 0.$$

Solving implicitly for K* and L*, the optimizing values

$$K^* = K^*(U, P, L, \bar{r})$$

$$L^* = L^*(U, P, K, w).$$

Let π^* be profit evaluated at K* and L* and let U* be utility associated with that value of profit. Then,

$$U^* = U(\pi^*), \text{ or}$$

$$U^* = U(PQ(K^*, L^*) - \bar{r}K^* - wL^*).$$

It can be seen that the response of U^* to an increase in $\bar{r}$ is negative since

$$\frac{\partial U^*}{\partial \bar{r}} = \frac{\partial U}{\partial \pi^*} \frac{\partial \pi^*}{\partial \bar{r}}$$

$$= -U'(\pi^*)K^* < 0.$$

This is true for all utility functions. The second derivative, however, is not uniquely defined.

$$\frac{\partial^2 U^*}{\partial \bar{r}^2} = U''(\pi^*)\left(\frac{\partial \pi^*}{\partial \bar{r}}\right)^2 + U'(\pi^*)\frac{\partial^2 \pi^*}{\partial \bar{r}^2}$$

$$= U''(\pi^*)(K^*)^2$$

$$- \frac{U'(\pi^*)}{P[Q^*_{KK} + 2Q_{KL}\frac{\partial L^*}{\partial K^*} + Q^*_{LL}\left(\frac{L^*}{K^*}\right)^2]}.$$

The results for the various utility functions are summarized as follows:

Risk Preference	$U''(\pi^*)$	$\frac{\partial^2 U^*}{\partial \bar{r}^2}$
Risk-taker	> 0	> 0
Risk-neutral	$= 0$	> 0
Risk-averse	< 0	$> 0, < 0.$[2]

Thus, U^* is a convex function of $\bar{r}$ for the risk-taking and risk-neutral firms. It may be convex for the risk-averse firm depending on the magnitude of $\frac{-U''(\pi^*)}{U'(\pi^*)}$, which is the Pratt-Arrow measure of absolute risk aversion. So, as in the previous analysis, the risk-neutral firm prefers uncertainty to certainty. The same is true for the risk-taking firm. The surprising result is the fact that there may be circumstances under which the risk-averse firm may prefer uncertainty. Not only is the utility function a factor here, but the price of the product and the characteristics of the production function may also influence the attitude of the firm toward the uncertainty.

Comparative Statics--The Uncertainty Case

Further contrasts and similarities between the traditional model and the uncertainty model may be ascertained by an analysis of

---

[1] If $\frac{-U''(\pi^*)}{U'(\pi^*)} \underset{(<)}{>} -\frac{1}{K^{*2}\, P[Q^*_{KK} + 2Q^*_{KL}\frac{\partial L^*}{\partial K^*} + Q^*_{LL}(\frac{\partial L^*}{\partial K^*})^2]}$

then $\frac{\partial^2 U^*}{\partial \bar{r}^2} \underset{(>)}{<} 0.$

the comparative statics of the model. Assume a shift (θ) in the mean of the distribution of r such that the variance remains unchanged. The new objective function becomes

$$\max_{K,L} Z = E[U(PQ(K,L) - (\tilde{r} + \theta)K - wL)].$$

where θ is the shift parameter.

The first-order conditions become

$$\frac{\partial Z}{\partial K} = E[U'(\pi)(PQ_K - (\tilde{r} + \theta))]$$

$$= 0. \tag{8}$$

$$\frac{\partial Z}{\partial L} = E[U'(\pi)(PQ_L - w)] = 0. \tag{9}$$

Equation (9) requires that

$$w = PQ_L,$$

since

$$E[U'(\pi)(PQ_L - w)] = E[U'(\pi)]\cdot[PQ_L - w]$$

and $E[U'(\pi)] > 0$.

Determination of the optimal response to a shift in the mean requires total differentiation of (8) and (9) with respect to θ and evaluating at $\theta = 0$. This yields

$$B_1\frac{\partial K}{\partial\theta} + A_2\frac{\partial L}{\partial\theta} - F_1 = 0$$

and

$$B_1\frac{\partial L}{\partial\theta} + A_1\frac{\partial K}{\partial\theta} - F_2 = 0$$

where

$$F_1 = E[U''(\pi)(PQ_L - w)(K)]$$

and

$$F_2 = E[U'(\pi) + U''(\pi)(PQ_K - \check{r})(K)].$$

Solving for $\frac{\partial K}{\partial\theta}$ and $\frac{\partial L}{\partial\theta}$ gives

$$\frac{\partial K}{\partial\theta} = \frac{A_2F_2 - B_1F_1}{D}$$

$$\frac{\partial L}{\partial\theta} = \frac{A_1F_1 - B_1F_2}{D}$$

A_1, A_2, B_1, and D have been defined in the previous section and

$$A_1 < 0,\ A_2 < 0,\ D > 0,$$

$$B_1 > 0, \text{ and } F_1 = 0.$$

It remains to determine the sign of F_2. To do this, make use of the Pratt-Arrow measure of absolute risk aversion. For the moment, assume

the firm is a risk averter and displays decreasing absolute risk aversion. That is,

$$U'(\pi) > 0,\ U''(\pi) < 0,\ R_A(\pi) = \frac{-U''(\pi)}{U'(\pi)} > 0$$

and $R_A'(\pi) < 0$. Let

$$\tilde{r} = PQ_K$$

and denote the profit at this level of $\tilde{r}$ as $\bar{\pi}$. $\bar{\pi}$ is nonstochastic. Then,

$$R_A(\pi) \geq R_A(\bar{\pi}) \text{ for } \tilde{r} \geq PQ_K$$

and

$$R_A(\pi) < R_A(\bar{\pi}) \text{ for } \tilde{r} < PQ_K$$

or

$$\frac{-U''(\pi)}{U'(\pi)} \geq R_A(\bar{\pi}) \qquad \tilde{r} \geq PQ_K \tag{10}$$

$$\frac{-U''(\pi)}{U'(\pi)} < R_A(\bar{\pi}) \qquad \tilde{r} < PQ_K. \tag{11}$$

We know that $-U'(\pi)(PQ_K - r) \geq 0$ for $\tilde{r} > PQ_K$ and $-U'(\pi)(PQ_K - \tilde{r}) < 0$ for $\tilde{r} < PQ_K$. Multiply (10) and (11) by $-U(\pi)(PQ_K - \tilde{r})$ obtaining

$$U''(\pi)(PQ_K - \tilde{r}) > -R_A(\bar{\pi})[U'(\pi)(PQ_K - \tilde{r})] \tag{12}$$

for all $\tilde{r}$. Taking the expected value of each side

$$E[U''(\pi)(PQ_K - \tilde{r})] > -R_A(\pi)\cdot E[U'(\pi)(PQ_K - \tilde{r})].$$

From (1), the right-hand side equals zero. Therefore,

$$E[U''(\pi)(PQ_K - \tilde{r})] > 0 \text{ for all } \tilde{r}.$$

This proof indicates that $F_2 > 0$. Consequently,

$$\frac{\partial K}{\partial \theta} < 0$$

and

$$\frac{\partial L}{\partial \theta} < 0.$$

Since,

$$\frac{\partial Q}{\partial \theta} = \frac{\partial Q}{\partial L}\frac{\partial L}{\partial \theta} + \frac{\partial Q}{\partial K}\frac{\partial K}{\partial \theta}$$

it follows that $\frac{\partial Q}{\partial \theta} < 0$. That is, given an increase in the mean of the distribution of the cost of capital, the optimal response is to decrease the use of capital and labor resulting in decreased output. This response is comparable to the certainty case.

Using a similar approach, one can determine the effect that

an increase in the variance of the distribution $f(\tilde{r})$ will have on the optimal allocation of capital and labor. Assume a mean preserving increase in risk. Rothschild and Stiglitz (1970) have shown that this necessarily implies increased variance. That is, the variance of the distribution increases, but the mean remains the same. In essence, one is analyzing the optimal response to an increase in uncertainty about the cost of capital.

Let

$$r^* = \gamma\tilde{r} + \theta$$

where $\gamma > 1$ and θ compensates for the effect of γ on $E[\tilde{r}]$. Also,

$$\frac{\partial\theta}{\partial\gamma} = -\mu$$

and

$$E[\tilde{r}] = \mu.$$

The objective function is now formulated as:

$$\underset{K,L}{\text{Max}}\ Z = E[U(PQ(K,L) - (\gamma\tilde{r} + \theta)K - wL)]$$

$$\frac{\partial Z}{\partial K} = E[U'(\pi)(PQ_K - (\gamma\tilde{r} + \theta))] = 0$$

$$\frac{\partial Z}{\partial L} = E[U'(\pi)(PQ_L - w)] = 0.$$

Totally differentiating with respect to γ and evaluating at

$$\gamma = 1$$

and

$$\theta = 0,$$

it can be shown that

$$A_1 \frac{\partial K}{\partial \gamma} + B_1 \frac{\partial L}{\partial \gamma} = H_2$$

and

$$B_1 \frac{\partial K}{\partial \gamma} + A_2 \frac{\partial L}{\partial \gamma} = H_1$$

where

$$H_1 = E[U''(\pi)(PQ_L - w)(\tilde{r} - \mu)(K)] = 0$$

and

$$H_2 = E[U''(\pi)(PQ_K - \tilde{r})(\tilde{r} - \mu)(K) + U'(\pi)(\tilde{r} - \mu)].$$

Furthermore, it follows that

$$\frac{\partial K}{\partial \gamma} = \frac{A_2 H_2 - B_1 H_1}{D}$$

and

$$\frac{\partial L}{\partial \gamma} = \frac{A_1 H_1 - B_1 H_2}{D}.$$

It can readily be seen that the signs of $\frac{\partial L}{\partial \gamma}$ and $\frac{\partial K}{\partial \gamma}$ depend on the sign of H_2. To determine the sign of H_2, one must determine the signs of $E[U'(\pi)(\tilde{r} - \mu)]$ and $E[U''(\pi)(PQ_K - \tilde{r})(\tilde{r} - \mu)(K)]$.

The first-order condition (1) can be rewritten as

$$E[U'(\pi)PQ_K] = E[U'(\pi)\tilde{r}]. \tag{13}$$

Subtract $E[U'(\pi)(\mu)]$ from each side of (13).

$$E[U'(\pi)(PQ_K - \mu)] = E[U'(\pi)(\tilde{r} - \mu)].$$

It is known that

$$PQ_K - E[\tilde{r}] = \frac{\text{cov}\ (U'(\pi), \tilde{r})}{E[U'(\pi)]}$$

a consequence of (13). For the risk-averter, the covariance term is positive; therefore,

$$PQ_K - E[\tilde{r}] > 0$$

and

$$E[U'(\pi)(PQ_K - \mu)] > 0.$$

Thus, $E[U'(\pi)(\tilde{r} - \mu)]$ is also greater than zero.

Now, rewrite $E[U''(\pi)(PQ_K - \tilde{r})(\tilde{r} - \mu)(K)]$ as $-KE[U''(\pi)(PQ_K - \tilde{r})(\mu - \tilde{r})]$. Adding and subtracting PQ_K, one obtains

$$-K \cdot E[U''(\pi)(PQ_K - \tilde{r})[(\mu - PQ_K) + (PQ_K - \tilde{r})]] = -K \cdot E[U''(\pi)(PQ_K - \tilde{r})(\mu - PQ_K) + U''(\pi)(PQ_K - \tilde{r})^2]$$

$$= -K \cdot E[U''(\pi)(PQ_K - \tilde{r})(\mu - PQ_K) + E[U''(\pi)(PQ_K - \tilde{r})^2]. \tag{14}$$

It has already been shown that $E[U''(\pi)(PQ_K - \tilde{r})] > 0$ and $(\mu - PQ_K) < 0$. And, since the second part of the expression is clearly negative, the entire expression is positive.

It follows, then, that,

$$\frac{\partial K}{\partial Y} < 0$$

and

$$\frac{\partial L}{\partial Y} < 0.$$

So, as in the case of an increase in mean, an increase in the variance of the cost of capital also leads to decreased utilization of both capital and labor and, consequently, decreased output.

Finally, as one might expect, the optimal response to an

increase in w, the contracted wage rate, is to decrease input usage. Totally differentiating the first-order conditions with respect to w and solving for $\frac{\partial K}{\partial w}$ and $\frac{\partial L}{\partial w}$ yields

$$A_1\frac{\partial L}{\partial w} + B_1\frac{\partial K}{\partial w} = C_1$$

$$= L\cdot E[U''(\pi)(PQ_L - w)] + E[U'(\pi)]$$

$$B_1\frac{\partial L}{\partial w} + A_2\frac{\partial K}{\partial w} = C_2$$

$$= L\cdot E[U''(\pi)(PQ_K - \tilde{r})].$$

All the signs needed have already been determined. Thus, it can be shown that

$$\frac{\partial K}{\partial w} < 0,$$

$$\frac{\partial L}{\partial w} < 0,$$

and

$$\frac{\partial Q}{\partial w} < 0.$$

Summary

The analysis presented here indicates that there are significant differences between the optimal allocation strategies for the firm under conditions of certainty and for the entrepreneur facing uncertainty. The first difference observed was that with respect to

the expansion path and the marginal rate of technical substitution. The firm with random input prices may not operate at its most efficient position and will be influenced by its degree of risk aversion. It was also shown that for the risk-taker, the risk-neutral firm, and for some risk-averse firms, profit is a convex function of the random input price. This implies that these producers prefer uncertainty. Thus, even though the firm may operate at less than full efficiency, it will still be in a position superior to the world of certainty.

Finally, the comparative statics were determined and found to be similar to the certainty case. The crucial assumption being that of decreasing absolute risk aversion. These firms react to changes in the distribution of the random input price as the firm under certainty would react to a change in the price.

CHAPTER IV
LABOR NEGOTIATIONS UNDER COST UNCERTAINTY

Introduction

The purpose of this chapter is to examine labor contract negotiations under the influence of uncertainty. The criteria for the existence of such contracts and the implications thereof are presented. Also, possible explanations for disagreement and/or strikes are proposed. The effects of an increase in variance, hence, an increase in uncertainty, are analyzed.

Labor Contract Model

The previous discussion has been based on the assumption that one factor price is random while the other is fixed. A possible explanation for the fixed wage rate is that a contract exists between the firm and labor. While in the long run the cost of labor is changing, the existence of the contract eliminates the uncertainty, at least in the short run.

Assume that both the cost of capital and the wage rate are random and symmetrically distributed. The problem of the firm may be formulated as

$$\max_{K,L} Z = E[U(P \cdot Q(K,L) - \tilde{r} \cdot K - \tilde{w} \cdot L)] .$$

The first-order conditions are

$$\frac{\partial Z}{\partial K} = E[U'(\pi)(P \cdot Q_K - \tilde{r})] = 0 \tag{15}$$

$$\frac{\partial Z}{\partial L} = E[U'(\pi)(P \cdot Q_L - \tilde{w})] = 0. \tag{16}$$

From (15) and (16), the optimal allocation of capital and labor must satisfy

$$\frac{E[U'(\pi)] \cdot E[\tilde{r}] + \mathrm{cov}(U'(\pi), \tilde{r})}{E[U'(\pi)] \cdot E[\tilde{w}] + \mathrm{cov}(U'(\pi), \tilde{w})} = \frac{Q_K}{Q_L}. \tag{17}$$

Assume the suppliers of the factor inputs have the same knowledge and expectations as the firm and are aware of the input decision of the firm. Further, assume one of them is willing to contract its supply of labor to the firm. What wage rate will the firm be willing to pay?

Substitute w*, a fixed wage rate, for w in the objective function and obtain (3).

$$\frac{Q_K}{Q_L} = \frac{E[U'(\pi)] \cdot E[\tilde{r}] + \mathrm{cov}(U'(\pi), \tilde{r})}{E[U'(\pi)]w^*}.$$

Assuming the same amount of labor is offered as the firm had originally intended to use, when its wage rate was random, enables one to equate (3) and (17). Assuming no decrease in labor utilization,

$$\frac{E[U'(\pi)]\cdot E[\tilde{r}] + \mathrm{cov}(U'(\pi),\ \tilde{r})}{E[U'(\pi)]\cdot E[\tilde{w}] + \mathrm{cov}(U'(\pi),\ \tilde{w})} = \frac{E[U'(\pi)]\cdot E[\tilde{r}] + \mathrm{cov}(U'(\pi),\ \tilde{r})}{E[U'(\pi)]\cdot w^*}. \tag{18}$$

It follows from (18) that

$$w^* - E[\tilde{w}] = \frac{\mathrm{cov}(U'(\pi),\ \tilde{w})}{E[U'(\pi)]}. \tag{19}$$

For the risk-averse firm, $\mathrm{cov}(U'(\pi),\ \tilde{w})$ is positive[3] and, therefore,

$$w^* - E[\tilde{w}] > 0.$$

This indicates that the firm would be willing to pay, for the same amount of labor, a wage rate higher than the expected value to eliminate uncertainty.

For the risk-taking firm, $\mathrm{cov}(U'(\pi),\ \tilde{w})$ is negative, hence,

$$w^* - E[\tilde{w}] < 0.$$

This is, the firm is willing to pay less than the expected value because it prefers uncertainty. In the risk-neutral case, $\mathrm{cov}(U'(\pi),\ \tilde{w})$ equals zero and w^* equals $E[w]$.

The rate w^*, however, is not necessarily the wage rate that would be agreed upon in the labor contract. To find an acceptable

---

[2]The sign of the covariance is found by evaluating the derivatives $\frac{\partial \tilde{w}}{\partial \tilde{w}}$ and $\frac{\partial U'(\pi)}{\partial \tilde{w}}$. $\frac{\partial w}{\partial \tilde{w}} = 1$, $\frac{\partial U'(\pi)}{\partial \tilde{w}} = U''(\pi)$, $\frac{\partial \pi}{\partial \tilde{w}} > 0$. Thus, $\mathrm{cov}(U'(\pi),\ \tilde{w}) > 0$.

rate, one must consider the risk characteristics of the labor supply. Let one develop a simple model in which the utility of labor (V) is a function only of the wage rate. One needs to solve for w_*, the lowest wage acceptable to the union. That is, find w_* such that

$$E[V(\tilde{w})] = V(w_*).$$

Since V is continuous and monotone increasing, V^{-1} exists, and

$$V^{-1}\{E[V(\tilde{w})]\} = V^{-1}V(w_*)$$

or

$$w_* = c \tag{20}$$

where

$$c = V^{-1}\{E[V(\tilde{w})]\}.$$

For the moment, consider the case where labor is risk averse ($V'' < 0$). Then, by Jensen's Inequality,

$$E[V(\tilde{w})] < V(E[\tilde{w}]),$$

so that

$$V^{-1}\{E[V(\tilde{w})]\} < V^{-1}\{V[E(w)]\}.$$

Finally, noticing that the left side of this expression is c,

$$c < E[w]. \tag{21}$$

Combining (20) and (21),

$$w_* < E[\tilde{w}].$$

Therefore, risk-averse labor would be willing to accept a wage less than the expected wage to avoid uncertainty. Although the magnitudes of w^* and w_* cannot be directly calculated, it can be seen that a range of acceptable rates exists. Refer to Figure 3.

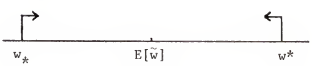

Figure 3

The actual wage agreed upon would depend on the bargaining powers of the respective parties to the contract.

Similar analysis shows that if one party is risk averse and the other risk neutral, mutually advantageous contract terms exist. If one party is risk neutral and the other a risk taker or if both are risk takers, then there will be no contract wage rate acceptable to both.

The case in which one party is risk averse and the other is a risk taker may or may not lead to a range of rates acceptable to both groups, depending upon the degree of risk aversion on the one side and risk preference on the other. Figure 4 represents one situation in

which an acceptable contract wage does exist.

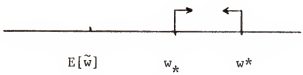

Figure 4

The assumption of risk aversion on the part of both bargaining parties may not be sufficient to ensure a mutually acceptable range if one relaxes the assumption of identical knowledge and expectations. Consider the case where the bargaining parties view different probability density functions. Specifically, they assess different means. The situation may then occur where no acceptable wage appears to exist (see Figure 5).

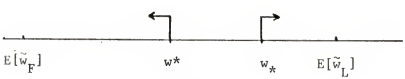

Figure 5

$E[\tilde{w}_F]$ and $E[\tilde{w}_L]$ are defined as the expected wage viewed by the firm and labor, respectively. But, differing assessments do not preclude the existence of an acceptable contract range (see Figure 6).

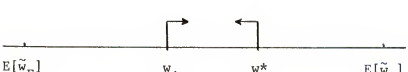

Figure 6

Another situation may arise as a result of one party's

incorrect perceptions of the other's utility function. In this case an acceptable range actually exists, but the magnitude of the range is erroneously assessed. Looking at this situation from the point of view of labor, they may offer a wage higher than the highest acceptable wage of the firm, but actually believe that it is within the range of acceptable wages. Refer to Figure 7.

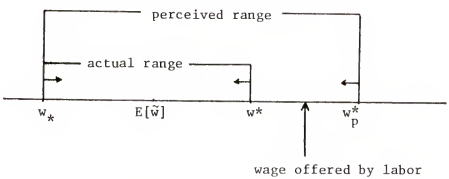

Figure 7

The perceived rate, w^*_p, is higher than the actual, w^*. Consequently, labor may make an offer unacceptable to the firm.

In both of these situations, however, one can expect a contract to be reached. The bargaining process and the information process, which are not mutually exclusive events, should help to alleviate these discrepancies.

Increased Risk and Contracts

The most interesting cases are the responses to increased variance. That is, how do the union and firm react to changes in the

variance of the distribution of wages, and what are the implications if the bargaining parties assess different variances? This situation has implications with regard to labor contracts consisting of a cost-of-living increase plus a merit increase. This contract may have the same expected wage rate as a fixed wage contract but is more accurately described as a risk situation in which the merit clause represents variability to the laborer. It will be shown that even a risk averter may prefer this situation depending on how he perceives his bargaining position.

Examining first the labor side, from a Taylor series expansion of $V(\tilde{w})$ about $\bar{w}$,

$$V(\tilde{w}) = V(\bar{w}) + (\tilde{w} - \bar{w})V'(\bar{w}) + (\tilde{w} - \bar{w})^2 \frac{V''(\bar{w})}{2} + (\tilde{w} - \bar{w})^3 \frac{V'''(\bar{w})}{6} + \phi$$

where

$$\phi = \text{higher-order terms.}$$

Assuming that ϕ is negligible and taking expectations of each side,

$$E[V(\tilde{w})] = V(E[\tilde{w}]) + E[(\tilde{w} - \bar{w})^2]\frac{V''(\bar{w})}{2}$$

or

$$E[V(\tilde{w})] = V(E[\tilde{w}]) + \sigma_w^{\,2}\,\frac{V''(\bar{w})}{2}^{2}\,. \tag{22}$$

Since $V''(\bar{w})$ is negative for the risk averter, it is easily seen that as $\sigma_w^{\,2}$ increases, $E[V(\tilde{w})]$ decreases.

$$\frac{\partial E[V(\tilde{w})]}{\partial \sigma w^2} = \frac{V''(\bar{w})}{2} < 0\,.$$

Recalling that w_*, the lowest wage acceptable to the union, is equal to $V^{-1}(E[V(\tilde{w})])$ and that $V(\tilde{w})$ is monotonically nondecreasing, w_* decreases as $\sigma_w^{\,2}$ increases. The proof follows from a well-known theorem in calculus (Apostol, 1964).

Theorem. Assume f is strictly increasing and continuous on an interval [a, b]. Let

$$c = f(a)$$

and

$$d = f(b)$$

and let g be the function obtained from f by inversion. That is, for each y in [c, d], let g(y) be that x in [a, b] such that

$$y = f(x).$$

Then,

1. g is strictly increasing on [c, d], and
2. g is continuous on [c, d].

Simply stated, if V is an increasing and continuous function, then V^{-1} is an increasing and continuous function. Therefore, if $E[V(\tilde{w})]$ is decreasing as σ_w^2 increases, $V^{-1}(E[V(\tilde{w})])$ or w_* is also decreasing. That is, the lowest acceptable wage for the union is even lower when the variance of the distribution of wages increases.

On the other side of the coin, the position of the firm, w*, is also affected by the increase in variance. Although the result is intuitively obvious, the mathematical proof is quite complex. Recall that

$$w^* - E[\tilde{w}] = \frac{\mathrm{cov}(U'(\pi), \tilde{w})}{E[U'(\pi)]}. \tag{23}$$

Since an increase in σ_w necessarily implies an increase in σ_w^2, $\frac{\partial w^*}{\partial \sigma_w}$ will be derived and evaluated as to its sign. Working with the denominator of (23) first, a Taylor series expansion of $U'(\pi)$ about $\bar{\pi}$ and taking expectations of each side yields

$$E[U'(\pi)] = U'(\bar{\pi}) + \sigma_{\pi}^{2} \frac{U'''(\bar{\pi})}{2} + \phi.$$

It will be assumed that ϕ is negligible. Since,

$$\pi = P \cdot Q - \tilde{r} \cdot K - \tilde{w} \cdot L,$$

$$\sigma_{\pi}^{2} = L^{2}\sigma_{w}^{2},$$

assuming σ_{w}^{2} and σ_{r}^{2} are independent and holding σ_{r}^{2} constant. So,

$$E[U'(\pi)] = U'(\pi) + L^{2}\sigma_{w}^{2} \frac{U'''(\pi)}{2}$$

From Pratt (1964), the assumption of decreasing absolute risk aversion implies $U'(\pi)U'''(\pi) \geq (U''(\pi))^{2}$, which in turn requires $U'''(\pi)$ to be positive. Therefore, $E[U'(\pi)]$ is an increasing function of σ_{w}^{2}.

The numerator can be expressed as

$$\text{cov}(U'(\pi), \tilde{w}) = \rho\sigma_{U'(\pi)}\sigma_{w}$$

where ρ is positive for the risk averter since it has already been shown that the covariance is positive for the case of risk aversion. To evaluate the effects of a change in $\sigma_{U'(\pi)}$ from a change in σ_{w}, $\sigma_{U'(\pi)}$ must be expressed in terms of σ_{w}. From the definition of variance,

$$\sigma_{U'(\pi)}^{2} = E[U'(\pi))^2] - \{E[U'(\pi)]\}^2.$$

The Taylor series expansion of $U'(\pi)$ about $\bar{\pi}$ yields

$$U'(\pi) = U'(\bar{\pi}) + (\pi - \bar{\pi})U''(\bar{\pi}) + (\pi - \bar{\pi})^2 \frac{U'''(\pi)}{2}, \tag{24}$$

ignoring higher order derivatives. Then,

$$E[U'(\pi)] = U'(\bar{\pi}) + \sigma_{\pi}^{2} \frac{U'''(\bar{\pi})}{2}$$

and

$$E[U'(\pi)]^2 = [U'(\bar{\pi}) + \sigma_{\pi}^{2} \frac{U'''(\bar{\pi})}{2}]^2.$$

Squaring (24) and taking the expected value of the result,

$$E[(U'(\pi))^2] = [U'(\bar{\pi})]^2 + \sigma_{\pi}^{2}[(U''(\bar{\pi}))^2 + 2U'(\bar{\pi}) \frac{U'''(\bar{\pi})}{2}] + 3\sigma_{\pi}^{2}\sigma_{\pi}^{2}[\frac{U'''(\bar{\pi})}{2}]^2$$

(See Appendix A for proof).

So,

$$\sigma_{U'(\pi)}^{2} = \sigma_{\pi}^{2}[(U''(\bar{\pi}))^2 + 2\sigma_{\pi}^{2}(\frac{U'''(\bar{\pi})}{2})^2]$$

and

$$\sigma_{U'(\pi)} = \sigma_{\pi}[(U''(\bar{\pi}))^2 + 2\sigma_{\pi}^2(\frac{U'''(\bar{\pi})}{2})^2]^{1/2}.$$

Thus,

$$\text{cov}(U'(\pi), \tilde{w}) = \rho\sigma_{\pi}[(U''(\bar{\pi}))^2 + 2\sigma_{\pi}^2(\frac{U'''(\bar{\pi})}{2})^2]^{1/2}\cdot\sigma_w. \tag{25}$$

Substituting

$$L^2\sigma_w^2 = \sigma_{\pi}^2$$

and

$$L\sigma_w = \sigma_{\pi}$$

in (25) and combining (22), (23), and (25), one obtains

$$w^* - E[w] = \frac{\rho L\sigma_w^2[(U''(\bar{\pi}))^2 + 2L^2\sigma_w^2(\frac{U'''(\bar{\pi})}{2})^2]^{1/2}}{U'(\bar{\pi}) + L^2\sigma_w^2\frac{U'''(\bar{\pi})}{2}}. \tag{26}$$

Differentiating with respect to σ_w, it can be shown that

$$\frac{\partial w^*}{\partial\sigma w} = \{(\sigma\frac{\partial\rho}{\partial\sigma})(L^3\sigma^3U''(\bar{\pi})^2\frac{U'''(\bar{\pi})}{2})$$

$$+ \left(\sigma\frac{\partial\rho}{\partial\sigma} + \rho\right)\left(2L^5\sigma^5 \frac{U'''(\bar{\pi})}{2}\right)^3$$

$$+ \left(\sigma\frac{\partial\rho}{\partial\sigma} + 2\rho\right)\left(L\sigma U'(\bar{\pi})U''(\bar{\pi})^2\right)$$

$$+ \left(\sigma\frac{\partial\rho}{\partial\sigma} + 3\rho\right)\left(2L^3\sigma^3\left(\frac{U'''(\bar{\pi})}{2}\right)^2\right)U'(\bar{\pi})\}$$

$$\div \{[(U''(\bar{\pi}))^2 + 2L^2\sigma^2\left(\frac{U'''(\bar{\pi})}{2}\right)^2]^{1/2}$$

$$\cdot [U'(\bar{\pi}) + L^2\sigma^2 \frac{U'''(\bar{\pi})}{2}]^2\}$$

where

$$\sigma = \sigma_w.$$

For $\rho > 0$, the following summarizes the sign of $\frac{\partial w^*}{\partial \sigma w}$.

$\frac{\partial\rho}{\partial\sigma w} < -3\frac{\rho}{\sigma w}$	$\frac{\partial w^*}{\partial\sigma w} < 0$
$0 > \frac{\partial\rho}{\partial\sigma w} > -3\frac{\rho}{\sigma w}$	$\frac{\partial w^*}{\partial\sigma w} \gtrless 0$
$\frac{\partial\rho}{\partial\sigma w} \geq 0$	$\frac{\partial w^*}{\partial\sigma w} \geq 0$

See Appendix B for proof.

For the linear and quadratic utility functions

$$\frac{\partial\rho}{\partial\sigma w} = 0$$

thus $\frac{\partial w^*}{\partial \sigma w}$ is positive. This means that as the variance of the distribution of wages increases, the maximum wage acceptable to the firm increases. Since w_* is simultaneously decreasing for the labor union, a wider bargaining range is defined.

This result may have several implications. First, the wider range of mutually acceptable wages may ease the bargaining process. It would be less likely that the bargaining parties would make an unacceptable offer. Second, if one of the bargaining parties is significantly more powerful than the other and is in a better bargaining position, that party is likely to demand and contract for a wage near or at the limit of the acceptable range. If that range is extended via an increase in variance, the more powerful party has the opportunity to better its position. Take, for example, the employee that is very accomplished in his work. This puts him in a better bargaining position than the average or below-average worker. He would enjoy an increase in variance as that affords him the opportunity to bargain for higher and higher wages. He would prefer a contract with a merit clause as that, in effect, increases the variance of the wage distribution. If the firm is the powerful participant, then it would prefer the increased variance as the opportunity for lower wage contracts increases. For other utility functions, the sign of $\frac{\partial w^*}{\partial \sigma w}$ is indeterminate. It depends upon $E[\pi]$ among other things. Also, the level of L, labor usage, has an impact

on $\frac{\partial w^*}{\partial \sigma_w}$.

Fixed Cost of Capital

Assume now that the cost of capital is fixed at $\bar{r}$. That is, the uncertainty is confined to one area of the firm, the cost of labor. How would this affect the maximum wage that the firm would be willing to pay for its labor? The procedure to analyze this is similar to previous examples.

The first-order conditions for $\bar{r}$ fixed and $\tilde{w}$ random can be found by appropriately modifying (4), resulting in

$$\frac{Q_K}{Q_L} = \frac{\bar{r}}{E[U'(\pi)]\cdot E[\tilde{w}] + \mathrm{cov}(U'(\pi), \tilde{w})}. \tag{27}$$

With $\bar{r}$ and $w^*_{\bar{r}}$ fixed, the condition becomes

$$\frac{Q_K}{Q_L} = \frac{\bar{r}}{w^*_{\bar{r}}}. \tag{28}$$

Equating (27) and (28), one solves for $w^*_{\bar{r}}$, the maximum contract wage given a fixed cost of capital and assuming the firm wishes to remain on its original expansion path:

$$w^*_{\bar{r}} = E[U'(\pi)]\cdot E[\tilde{w}] + \mathrm{cov}(U'(\pi), \tilde{w}). \tag{29}$$

From (14), one knows the maximum contract wage given random cost of capital as

$$w^* = \frac{E[U'(\pi)]\cdot E[\tilde{w}] + \mathrm{cov}(U'(\pi), \tilde{w})}{E[U'(\pi)]}.$$

Comparing $w_{\bar{r}}^*$ and w^*

$$w^* = \frac{w_{\bar{r}}^*}{E[U'(\pi)]}. \tag{30}$$

If $E[U'(\pi)] > 1$, this indicates that $w_{\bar{r}}^* > w^*$. Conversely, if $E[U'(\pi)] < 1$, then $w_{\bar{r}}^* < w^*$. Thus, one cannot uniquely specify the relationship. However, given no change in the utility function of the firm, the behavior of the risk evader may change as profits increase. That is, the expected slope decreases as profits increase and, consequently, the relationship between w^* and $w_{\bar{r}}^*$ also changes. When the expected slope is greater than 1, the attitude of the firm toward uncertainty is emphasized in the form of $w_{\bar{r}}^*$ being larger than w^*. As profits increase and $E[U'(\pi)]$ becomes less than 1, the reverse is true. The maximum contract wage of the firm will be less than the case with uncertain cost of capital.

Summary

It has been shown that the introduction of uncertainty and the

assumption of expected utility maximization has a significant effect on the positions the firm and labor may take during wage negotiations. The ability to accurately assess the probability distribution of the wage rate and the utility function of the opposing party is essential for a bargaining range to be defined. Absence of this range may lead to a strike and/or forced arbitration. The actual outcome of the negotiations is dependent on the bargaining powers and behaviors of the individual parties. The more powerful bargainer actually enjoys the uncertainty that exists as it increases the range of acceptable wages and that party can then take advantage of the risk aversion of its opponent.

CHAPTER V
THE DUOPOLIST UNDER COST UNCERTAINTY

Introduction

The first oligopoly model was developed by Augustin Cournot in 1838. Cournot's argument concerned the behavior of two firms in a duopolistic world. Each produced an identical product. The price of the product was determined by the combined output of both duopolists. The basic behavioral characteristics which Cournot ascribed to the competitors were that each duopolist acted as profit maximizers under the assumption that the quantity produced by his rival was invariant with respect to his own production level. The results of this behavior lead the duopolists to a stable equilibrium output at which there is no incentive to further change their production level. This output is

$$Q_i = \frac{MC - P}{\frac{\partial P}{\partial Q_i}}$$

$$i = 1,2$$

where MC is the marginal cost. Furthermore, Cournot showed that as the number of firms approaches infinity, the behavior of the firms approaches that of a purely competitive industry.

This chapter extends the analysis of Cournot to examine the expected utility maximizer under conditions of cost uncertainty.

The model presented assumes a duopolistic industry where the firms are naive quantity setters. Initially, the firms are assumed to have the same utility functions and the same expectations with regard to the distribution function of cost. These assumptions, however, are relaxed later.

Cournot Model under Cost Uncertainty

Define profit as

$$\pi_1 = P \cdot Q_1 - \tilde{c} \cdot Q_1$$

where

$$P = f(Q)$$

is a negatively sloped demand function,

$$Q = Q_1 + Q_2$$

and $\tilde{c}$ is a random cost (nonnegative). Then, the naive firm 1, which believes firm 2 will maintain its present output faces the problem:

$$\max_{Q_1} E[U_1(\pi_1)] = E[U_1(P \cdot Q_1 - \tilde{c} \cdot Q_1)].$$

At the maximum,

$$\frac{\partial E[U_1(\pi_1)]}{\partial Q_1} = E[U_1'(\pi_1)(\frac{\partial P}{\partial Q_1}\cdot Q_1 + P - \tilde{c})]$$

$$= 0 \tag{31}$$

and,

$$\frac{\partial^2 E[U_1(\pi_1)]}{\partial Q_1} = E[U_1''(\pi_1)(\frac{\partial P}{\partial Q_1}\cdot Q_1 + P - \tilde{c})^2]$$

$$+ E[U_1'(\pi_1)(\frac{\partial^2 P}{\partial Q_1^2}\cdot Q_1 + 2\frac{\partial P}{\partial Q_1})]$$

which must be negative. For the risk-neutral firm or the risk-averse firm,

$$\frac{\partial^2 P}{\partial Q_1^2} < 0$$

ensures optimality. The risk taker will not be considered in this chapter.

Return to (31), solving for Q_1

$$Q_1 = \frac{E[U_1'(\pi_1), \tilde{c}] - E[U_1'(\pi_1)]\cdot P}{E[U'(\pi_1)]\frac{\partial P}{\partial Q_1}}$$

$$+ \frac{E[U_1'(\pi_1)]\ E[\tilde{c}] + \mathrm{cov}(U_1'(\pi_1),\ \tilde{c}) - E[U_1'(\pi_1)]\ P}{E[U_1'(\pi_1)]\frac{\partial P}{\partial Q_1}}$$

$$Q_1 = \frac{E[\tilde{c}] - P}{\frac{\partial P}{\partial Q_1}} + \frac{\mathrm{cov}(U_1'(\pi_1),\ \tilde{c})}{E[U_1'(\pi_1)]\ \frac{\partial P}{\partial Q_1}}. \tag{32}$$

Consider the case where $\tilde{c}$ is found at $E[\tilde{c}]$ then,

$$Q_1 = \frac{E[\tilde{c}] - P}{\frac{\partial P}{\partial Q_1}}.$$

This is identical to the traditional Cournot model. Similarly, if the firm is risk neutral, $U_1'(\pi_1)$ is a constant, and once again the result is the same as the certainty case. However, if the firm is risk averse, then,

$$\mathrm{cov}(U_1'(\pi_1),\ \tilde{c}) > 0,$$

$$E[U_1'(\pi_1)] > 0,$$

and since

$$\frac{\partial P}{\partial Q_1} < 0,$$

the optimal quantity produced is less than the risk-neutral firm and, consequently, the output of the industry is reduced if there exists at least one risk-averse firm. This proposition will be further

analyzed under various risk-preference assumptions.

Considering the duopoly case, firm 2 will produce Q_2,

$$Q_2 = \frac{E[\tilde{c}] - P}{\frac{\partial P}{\partial Q_2}} + \frac{\mathrm{cov}(U_2'(\pi_2), \tilde{c})}{E[U_2'(\pi_2)] \frac{\partial P}{\partial Q_2}}. \tag{33}$$

Since the firms face the same demand function, $f(Q)$, where

$$\frac{\partial P}{\partial Q_2} = \frac{\partial P}{\partial Q_1}$$

(32) and (33) may be written

$$Q_i = \frac{E[\tilde{c}] - f(Q)}{f'(Q)} + \frac{\mathrm{cov}(U_i'(\pi_i), \tilde{c})}{E[U_i'(\pi_i)]f'(Q)}$$

for

$$i = 1,2. \tag{34}$$

Noting also that

$$Q = Q_1 + Q_2,$$

(34) describes Q_i as a function of the output of the rest of the industry, in this case, the other firm. This function is the reaction curve. If each firm produces according to its reaction curve, they will produce where the reaction curves intersect. This point is the Cournot equilibrium point and, if the system is stable, there will be

no incentive to move away from this point and, thus, the output for each firm and for the industry will remain constant. The equilibrium will be stable if

$$0 > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 1} > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 2.}$$

Linear Demand Function

For expository purposes, assume a linear industry demand function and risk-neutral firms.

$$P = a + b(Q_1 + Q_2)$$

where $b < 0$

$$\frac{\partial P}{\partial Q} = b.$$

Thus, the reaction curves for the first two figures become

$$Q_1 = \frac{E[\tilde{c}] - a - b(Q_1 + Q_2)}{b}$$

$$Q_2 = \frac{E[\tilde{c}] - a - b(Q_1 + Q_2)}{b}.$$

Solving for Q_1 in terms of Q_2 and Q_2 in terms of Q_1,

$$Q_1 = \frac{E[\tilde{c}] - a - bQ_2}{2b}$$

or

$$Q_1 = \frac{E[\tilde{c}] - a}{2b} - \frac{Q_2}{2}$$

and

$$Q_2 = \frac{E[\tilde{c}] - a}{2b} - \frac{Q_1}{2}$$

Figure 8 shows the reaction curves and the equilibrium position.

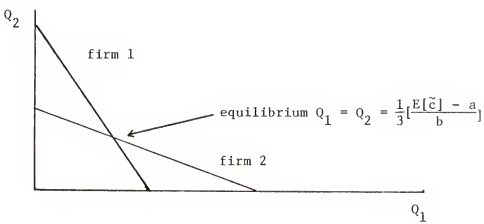

Figure 8

Since Q_1 and $Q_2 \geq 0$, $E[\tilde{c}] - a < 0$. The equilibrium is stable since

$$(\frac{\partial Q_1}{\partial Q_2})\text{firm 1} = -1/2$$

and

$$\left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 2} = -2.$$

Thus,

$$0 > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 1} > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 2}.$$

Total output is given by

$$Q_1 + Q_2 = \frac{2}{3}\left[\frac{E[\tilde{c}] - a}{b}\right].$$

Suppose, however, firm 1 is risk averse. Then,

$$Q_1 = \frac{E[\tilde{c}] - a - b(Q_1 + Q_2)}{b} + \frac{\mathrm{cov}(U_1'(\pi_1), \tilde{c})}{E[U_1'(\pi_1)]\, b}$$

or

$$Q_1 = \left[\frac{E[\tilde{c}] - a}{2b} + \frac{\mathrm{cov}(U_1'(\pi_1), \tilde{c})}{2bE[U_1'(\pi_1)]}\right] - \frac{Q_2}{2}$$

and

$$Q_2 = \frac{E[\tilde{c}] - a}{2b} - \frac{Q_1}{2}.$$

The equilibrium output of firm 1 decreases and the output of firm 2

increases as long as

$$\frac{E[\tilde{c}] - a}{b} + \frac{\mathrm{cov}(U_1'(\pi_1), \tilde{c})}{bE[U_1'(\pi_1)]} > \frac{E[\tilde{c}] - a}{2b}$$

See Figure 9.

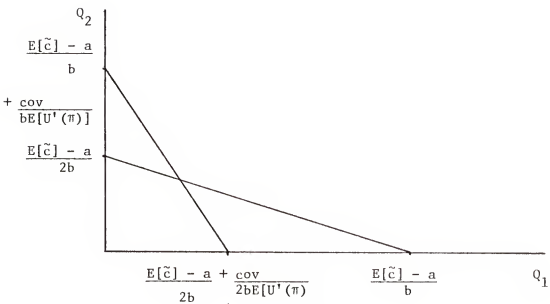

Figure 9

Total industry output decreases as

$$Q_1 + Q_2 = \frac{2}{3}\left[\frac{E[\tilde{c}] - a}{b} + \frac{\mathrm{cov}(U_1'(\pi_1), \tilde{c})}{2bE[U_1'(\pi_1)]}\right].$$

If firm 1 is risk averse to the extent that

$$\frac{\mathrm{cov}(U_1'(\pi_1),\ \tilde{c})}{E[U_1'(\pi_1)]} \geq \frac{a - E[\tilde{c}]}{2}$$

then firm 1 will produce nothing and firm 2 will produce

$$Q_2 = \frac{E[\tilde{c}] - a}{2b}.$$

This system is also stable. Although the slope of the reaction curve of firm 1 is not constant, it can be shown that the conditions for stability are met.

$$\left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 2} = -2.$$

For firm 1,

$$\frac{Q_1}{Q_2} = \frac{\{-\mathrm{cov}(U_1'(\pi_1),\ \tilde{c})[2b\frac{\partial EU_1'(\pi_1)}{\partial Q}]\} \div D^2 - 1/2}{1 + \{\mathrm{cov}(U_1'(\pi_1),\ \tilde{c})[2b\frac{\partial EU_1'(\pi_1)}{\partial Q}]\} \div D^2} \tag{35}$$

where

$$D = \{2bE[U_1'(\pi_1)]\}^2.$$

This slope is negative. Let

$$C = \mathrm{cov}(U_1'(\pi_1),\ \tilde{c})[2b\frac{\delta EU_1'(\pi_1)}{\delta Q}]$$

then (35) may be rewritten

$$\frac{\frac{-C}{D^2} - \frac{1}{2}}{1 + \frac{C}{D^2}}.$$

Since

$$\frac{C}{D^2} > 0,$$

$$-\frac{C}{D^2} - 1/2 > \frac{C}{D^2} - 1. \tag{36}$$

Also,

$$-\frac{C}{D^2} - 1 > 2(-\frac{C}{D^2} - 1). \tag{37}$$

Thus, from (36) and (37)

$$-\frac{C}{D^2} - 1/2 > 2(-\frac{C}{D} - 1)$$

or

$$\frac{-C}{D^2} - 1/2 > -2(\frac{C}{D} + 1),$$

and finally

$$\frac{\frac{-C}{D^2} - 1/2}{1 + C/D^2} > -2.$$

So,

$$0 > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 1} > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 2}.$$

Thus, the Cournot equilibrium exists and is stable.

Finally, assume both firm 1 and firm 2 are equally risk averse. Then,

$$Q_1 = \left[\frac{E[\tilde{c}] - a}{2b} + \frac{\text{cov}(U_1'(\pi_1), \tilde{c})}{2bE[U_1'(\pi_1)]}\right] - \frac{Q_2}{2}$$

and

$$Q_2 = \left[\frac{E[\tilde{c}] - a}{2b} + \frac{\text{cov}(U_2'(\pi_2), \tilde{c})}{2bE[U_2'(\pi_2)]}\right] - \frac{Q_1}{2}.$$

These are the reaction curves. Total industry output

$$Q_1 + Q_2 = \frac{2}{3}\left[\frac{E[\tilde{c}] - a}{b} + \frac{\text{cov}(U'(\pi), \tilde{c})}{bE[U'(\pi)]}\right].$$

As the second term is negative, the amount produced by the industry will decrease once again from the previous cases. Each firm will produce

$$1/3\left[\frac{E[\tilde{c}] - a}{b} + \frac{\text{cov}(U'(\pi), \tilde{c})}{bE[U'(\pi)]}\right].$$

Since the utility functions were assumed to be equal, no differentiation need be made between firm 1 and firm 2 in terms of U or π.

Once again the Cournot equilibrium can be shown to be stable.

$$\left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 1 = \frac{\{-\text{cov}(U'(\pi),\ \tilde{c})[2b\frac{\partial EU'(\pi)}{\partial Q}]\} \div D^2 - 1/2}{1 + \{\text{cov}(U'(\pi),\ \tilde{c})[2b\frac{\partial EU'(\pi)}{\partial Q}]\} \div D^2}$$

$$= \frac{\frac{-C}{D^2} - \frac{1}{2}}{1 + \frac{C}{D^2}} > 0$$

and

$$\left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 2 = \frac{1 + \frac{C}{D^2}}{\frac{-C}{D^2} - \frac{1}{2}}.$$

Let

$\frac{C}{D^2} = A$, for simplicity; $A > 0$, then,

$$-\frac{(A + 1/2)}{(A + 1)} = \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 1$$

$$-\frac{(A + 1)}{(A + 1/2)} = \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 2.$$

Now one needs to determine the magnitudes.

$$-\frac{(A + 1/2)}{(A + 1)} \left(\gtreqless\right) -\frac{(A + 1)}{(A + 1/2)}$$

$$- (A + 1/2)(A + 1/2) \ (\gtreqless) - (A + 1)(A + 1)$$

$$- [A^2 + A + 1/4] \ (\gtreqless) - [A^2 + 2A + 1]$$

$$- [A + 1/4] \ (\gtreqless) - [2A + 1]$$

$$- 1/4 > - 1 - A$$

Thus, since $A > 0$,

$$0 > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 1} > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 2.}$$

General Demand Function

Let

$$P = f(Q)$$

where

$$Q = Q_1 + Q_2$$

and

$$\frac{\partial P}{\partial Q_1} < 0, \ \frac{\partial P}{\partial Q_2} < 0, \ \frac{\partial P}{\partial Q} < 0.$$

Then, the optimal production for the duopolist firms can be expressed as

$$Q_i = \frac{E[\tilde{c}] = f(Q)}{f'(Q)} + \frac{\mathrm{cov}(U_i'(\pi_i), \tilde{c})}{E[U_i'(\pi_i)] \cdot f'(Q)}$$

$$i = 1,2. \tag{38}$$

Since Q_1 and $Q_2 \geq 0$, the reaction curves for the firms have positive intercepts. It can also be shown that the slopes are negative and a Cournot equilibrium exists and is stable. Equation (38) can be expressed

$$Q_1 = g(Q) + h(Q)$$

where

$$Q = Q_1 + Q_2$$

or

$$g(Q) + h(Q) - Q_1 = 0.$$

Differentiating with respect to Q_2,

$$g'(Q)[\frac{\partial Q}{\partial Q_1}\frac{\partial Q_1}{\partial Q_2} + \frac{\partial Q}{\partial Q_2}\frac{\partial Q_2}{\partial Q_2}]$$

$$h'(Q)[\frac{\partial Q}{\partial Q_1}\frac{\partial Q_1}{\partial Q_2} + \frac{\partial Q}{\partial Q_2}\frac{\partial Q_2}{\partial Q_2}] - \frac{\partial Q_1}{\partial Q_2} = 0.$$

Solving for $\frac{\partial Q_1}{\partial Q_2}$ yields

$$\frac{\partial Q_1}{\partial Q_2} = \frac{-\ (g'(Q) + h'(Q))}{(g'(Q) + h'(Q) - 1}.$$

Letting

$$\frac{E[\tilde{c}] - f(Q)}{f'(Q)} = g(Q)$$

and

$$\frac{\mathrm{cov}(U_1'(\pi_1),\ \tilde{c})}{E[U_1'(\pi_1)]f'(Q)} = h(Q)$$

then,

$$g'(Q) = \frac{-f'(Q) \cdot f'(Q) - [E[\tilde{c}] - f(Q)]f''(Q)}{[f'(Q)]^2}$$

and

$$h'(Q) = \frac{-\mathrm{cov}(U_1'(\pi_1),\ \tilde{c})\ [\frac{\partial E[U_1'(\pi_1)]}{\partial Q} \cdot f'(Q) + E[U_1'(\pi_1)]f''(Q)]}{[E[U_1'(\pi_1)]f'(Q)]^2}.$$

Then, $g'(Q) + h'(Q)$ can be written,

$$\begin{aligned} g'(Q) + h'(Q) = \{&-f'(Q)^2 E[U_1'(\pi_1)]^2 \\ &- [Ec[\tilde{c}] - f(Q)]f''(Q)E[U_1'(\pi_1)]^2 \\ &-\mathrm{cov}(U_1'(\pi_1),\ \tilde{c}) \end{aligned}$$

$$[\frac{\partial E[U_1'(\pi_1)]}{\partial Q} f(Q) + E[U_1'(\pi_1)]f''(Q)]\}$$

$$\div [E[U_1'(\pi_1)]f'(Q)]^2$$

$$= \{-f(Q)^2 E[U_1'(\pi_1)]^2 - \{[E[\tilde{c}]$$

$$- f(Q)]E[U_1'(\pi_1)] + \mathrm{cov}(U_1'(\pi_1), \tilde{c})\}$$

$$\cdot f''(Q) \cdot E[U_1'(\pi_1)]$$

$$- \mathrm{cov}(U_1'(\pi_1), \tilde{c})\frac{\partial E[U_1'(\pi_1)]}{\partial Q} f(Q)\}$$

$$\div E[U_1'(\pi_1)]^2 f'(Q)^2.$$

One knows the signs of all terms except

$$\{[E[c] - f(Q)]E[U_1'(\pi_1)] + \mathrm{cov}(U_1'(\pi_1), \tilde{c})\}.$$

Return to (38)

$$Q_1 = \frac{E[c] - f(Q)}{f'(Q)} + \frac{\mathrm{cov}(U_1'(\pi_1), \tilde{c})}{E[U_1'(\pi_1)] \quad f'(Q)}$$

$$\geq 0.$$

Since $f'(Q) < 0$,

$$E[\tilde{c}] = f(Q) + \frac{\mathrm{cov}(U_1'(\pi_1), \tilde{c})}{E[U_1'(\pi_1)]} \leq 0$$

and, thus,

$$[E[\tilde{c}] - f(Q)] \cdot E[U_1'(\pi_1)]$$

$$+ \operatorname{cov}(U_1'(\pi_1), \tilde{c}) \leq 0.$$

Therefore, $g'(Q) + h'(Q) < 0$, and since

$$\frac{\partial Q_1}{\partial Q_2} = \frac{-(g'(Q) + h'(Q))}{g'(Q) + h'(Q) - 1},$$

$$\left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 1 < 0.$$

Now, observing that

$$\left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 2 = \frac{1}{\left(\frac{\partial Q_2}{\partial Q_1}\right)\text{firm } 2}$$

and that

$$\left(\frac{\partial Q_2}{\partial Q_1}\right)\text{firm } 2 = \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 1,$$

it can be seen that

$$\left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 2 = \frac{-(g'(Q) + h'(Q) - 1)}{(g'(Q) + h'(Q))} < 0.$$

It remains to show that

$$\left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 1 > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm } 2.$$

or that

$$\frac{-(g'(Q) + h'(Q))}{g'(Q) + h'(Q) - 1} > \frac{-(g'(Q) + h'(Q) - 1)}{(g'(Q) + h'(Q))}$$

Let

$$x = g'(Q) + h'(Q) < 0$$

then,

$$\frac{-x}{(x - 1)} \gtreqqless \frac{-(x - 1)}{x}$$

or

$$-x^2 \ (\gtreqqless) \ -(x - 1)^2$$

$$-x^2 \ (\gtreqqless) \ -(x^2 - 2x + 1)$$

$$-x^2 \ (\gtreqqless) \ -x^2 + (2x - 1)$$

since $x < 0$

$$-x^2 > -x^2 + (2x - 1)$$

and, therefore,

$$0 > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 1} > \left(\frac{\partial Q_1}{\partial Q_2}\right)\text{firm 2.}$$

Thus, with a general demand function, the Cournot equilibrium is stable.

One can now examine this equilibrium under various risk preferences. Suppose both firm 1 and firm 2 are risk neutral. Then, since the covariance terms are zero

$$Q_1 = \frac{E[\tilde{c}] - f(Q)}{f'(Q)} = Q_2$$

and

$$Q_1 + Q_2 = [\frac{E[\tilde{c}] - f(Q)}{f'(Q)}] \cdot 2.$$

If firm 1 is risk averse, then

$$Q_1 = \frac{E[\tilde{c}] - f(Q)}{f'(Q)} + \frac{\mathrm{cov}(U_1'(\pi_1), \tilde{c})}{E[U_1'(\pi_1)]\, f'(Q)}.$$

The second term is negative, so the output of firm 1 will decrease. For firm 2,

$$Q_2 = \frac{E[\tilde{c}] - f(Q)}{f'(Q)}.$$

Total industry output is given by

$$Q_1 + Q_2 = 2[\frac{E[\tilde{c}] - f(Q)}{f'(Q)}] + \frac{\mathrm{cov}(U_1'(\pi_1), \tilde{c})}{E[U_1'(\pi_1)]\, f'(Q)}$$

which is also less than the case where both firms were risk neutral.

Finally, if both firms are risk averse, then, output will be

even smaller. If both are equally risk averse, then

$$Q_1 = Q_2;$$

however, if one is more risk averse, then that firm will produce less than its competitor.

Comparative Statics

The optimal responses to a change in the mean or variance of the distribution of cost can be assessed for the Cournot model. First, assume a shift θ in the mean such that the new profit function becomes

$$E[U(\pi)] = E[U(P \cdot Q - (\tilde{c} + \theta)Q].$$

The first-order condition is

$$\frac{\partial E[U(\pi)]}{\partial Q} = E[U'(\pi)\ \frac{\partial P}{\partial Q} \cdot Q + P - (\tilde{c} + \theta)] = 0.$$

Total differentiation with respect to θ yields,

$$E[U''(\pi)(\frac{\partial P}{\partial Q}\frac{\partial Q}{\partial \theta} \cdot Q + P\frac{\partial Q}{\partial \theta}$$

$$- (\tilde{c} + \theta)\frac{\partial Q}{\partial \theta} - Q)(\frac{\partial \pi}{\partial Q})$$

$$+ U'(\pi)((\frac{\partial^2 P}{\partial Q^2} \cdot Q + 2\frac{\partial P}{\partial Q})\frac{\partial Q}{\partial \theta} - 1)] = 0.$$

Evaluating at

$$\theta = 0$$

and solving for $\frac{\partial Q}{\partial \theta}$,

$$\frac{\partial Q}{\partial \theta} = \frac{Q \cdot E[U''(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})] + E[U'(\pi)]}{E[U''(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})^2] + E[U'(\pi)(\frac{\partial^2 P}{\partial Q^2} \cdot Q + 2\frac{\partial P}{\partial Q})]}. \tag{39}$$

The denominator of (39) is the second-order condition and is negative. $E[U'(\pi)]$ and Q are positive. Thus, the expression of concern is

$$E[U''(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})].$$

Assume the firm is a risk averter and that its utility function has the property of decreasing absolute risk aversion. That is,

$$R_A(\pi) = \frac{-U''(\pi)}{U'(\pi)} > 0$$

and $R_A'(\pi) < 0$. Let

$$\tilde{c} = (\frac{\partial P}{\partial Q} \cdot Q + P)$$

and denote profit at this level of $\tilde{c}$ as $\bar{\pi}$. Then,

$$R_A(\pi) \geq R_A(\bar{\pi}) \text{ for } \tilde{c} \geq (\frac{\partial P}{\partial Q} \cdot Q + P)$$

and

$$R_A(\pi) < R_A(\bar{\pi}) \text{ for } \tilde{c} < (\frac{\partial P}{\partial Q}) \cdot Q + P)$$

or

$$\frac{-U''(\pi)}{U'(\pi)} \geq R_A(\bar{\pi}) \text{ for } \tilde{c} \geq (\frac{\partial P}{\partial Q} \cdot Q + P) \quad (40)$$

$$\frac{-U''(\pi)}{U'(\pi)} < R_A(\bar{\pi}) \text{ for } \tilde{c} < (\frac{\partial P}{\partial Q} \cdot Q + P). \quad (41)$$

Since $U'(\pi) > 0$, one knows that

$$-U'(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c}) \geq 0 \text{ for } \tilde{c} \geq (\frac{\partial P}{\partial Q} \cdot Q + P),$$

and

$$-U'(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c}) < 0 \text{ for } \tilde{c} < (\frac{\partial P}{\partial Q} \cdot Q + P).$$

Multiply (40) and (41) by

$$-U'(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})$$

obtaining

$$U''(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c}) > -R_A(\bar{\pi})[U'(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})] \quad (42)$$

for all $\tilde{c}$. Taking the expected value of each side of (42) yields

$$E[U''(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})] > -R_A(\bar{\pi}) \cdot E[U'(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})].$$

From (31), the right-hand side is zero. Thus,

$$E[U''(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})] > 0 \text{ for all } \tilde{c} \tag{43}$$

and (39) is negative. Therefore,

$$\frac{\partial Q}{\partial \theta} < 0.$$

That is, if the mean of the distribution increases, the optimal response of the firm is to decrease output.

Now assume a mean preserving increasing in the variance of the distribution. Let

$$c^* = \gamma \cdot \tilde{c} + \theta$$

where

$$\gamma > 1, \frac{\partial \theta}{\partial \gamma} = -\mu$$

and

$$E[\tilde{c}] = \mu.$$

The objective becomes,

$$\text{Max } E[U(\pi)] = E[U(P \cdot Q - (\gamma\tilde{c} + \theta)Q)]$$

and the first-order condition is

$$\frac{E[U(\pi)]}{\partial Q} = E[U'(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - (\gamma\tilde{c} + \theta))]$$

$$= 0.$$

Total differentiation with respect to γ and evaluating at

$$\gamma = 1$$

and

$$\theta = 0$$

yields

$$E[U''(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})^2]\frac{\partial Q}{\partial \gamma} - E[U''(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})(\tilde{c} + \mu)Q]$$

$$+ E[U'(\pi)(\frac{\partial^2 P}{\partial Q^2} \cdot Q + 2\frac{\partial P}{\partial Q})]\frac{\partial Q}{\partial \gamma} - E[U'(\pi)(\tilde{c} + \mu)].$$

Solving for $\frac{\partial Q}{\partial \gamma}$,

$$\frac{\partial Q}{\partial \gamma} = \frac{Q \cdot E[U''(\pi)(\frac{\partial P}{\partial Q} \cdot Q + P - \tilde{c})(\tilde{c} + \mu)] + E[U'(\pi)(\tilde{c} + \mu)]}{D} \tag{44}$$

where D is the second-order condition and is negative. In order to ascertain the sign of the numerator, the first-order condition must be expressed as

$$E[U'(\pi)(Q\frac{\partial P}{\partial Q} + P)] = E[U'(\pi)(\tilde{c})]. \tag{45}$$

Adding $E[U'(\pi)(\mu)]$ to each side yields

$$E[U'(\pi)](Q\frac{\partial P}{\partial Q} + P + \mu) = E[U'(\pi)(\tilde{c} + \mu)]. \tag{46}$$

From (45), one can write

$$E[U'(\pi)](Q\frac{\partial P}{\partial Q} + P) = E[U'(\pi)] \cdot E[\tilde{c}] + cov(U'(\pi), \tilde{c})$$

or

$$Q\frac{\partial P}{\partial Q} + P - E[\tilde{c}] = \frac{cov(U'(\pi), \tilde{c})}{E[U'(\pi)]}. \tag{47}$$

Since

$$E[\tilde{c}] = -\mu$$

and the covariance term is positive for the risk-averse firm,

$$Q\frac{\partial P}{\partial Q} + P + \mu > 0. \tag{48}$$

Thus,

$$E[U'(\pi)](Q\frac{\partial P}{\partial Q} + P + \mu) > 0.$$

And from (46)

$$E[U'(\pi)(\tilde{c} + \mu)] > 0.$$

Now working with the first part of the numerator of (44), it can be written as

$$- Q \cdot E[U''(\pi)(Q\frac{\partial P}{\partial Q} + P - \tilde{c})(-\tilde{c} - \mu)]. \tag{49}$$

Add and subtract

$$(Q\frac{\partial P}{\partial Q} + P)$$

to (49) obtaining

$$- Q\{E[U''(\pi)(Q\frac{\partial P}{\partial Q} + P - \tilde{c})^2] + E[U''(\pi)(-\mu - Q\frac{\partial P}{\partial Q} - P)$$

$$(Q\frac{\partial P}{\partial Q} + P - \tilde{c})]\}.$$

One knows from (43) that

$$E[U''(\pi)(Q\frac{\partial P}{\partial Q} + P - \tilde{c})] > 0$$

and that

$$(-\mu - Q\frac{\partial P}{\partial Q} - P) = -(Q \cdot \frac{\partial P}{\partial Q} + P + \mu)$$

which from (48) is negative. Thus,

$$-Q \cdot E[U''(\pi)(-\mu - Q \frac{\partial P}{\partial Q} - P)(Q \cdot \frac{\partial P}{\partial Q} + P - \tilde{c})] > 0$$

and

$$-Q \cdot E[U''(\pi)(Q \cdot \frac{\partial P}{\partial Q} + P - \tilde{c})^2] > 0$$

since $E[U''(\pi)] < 0$ for risk-averse firms. Therefore, the numerator of (44) is positive and the denominator is negative. It follows that

$$\frac{\partial Q}{\partial \gamma} > 0.$$

The optimal response to an increase in variance is to decrease output.

Differing Cost Expectations

This last section will look at the effects of differing expectations of the mean of the distribution of costs. Assume the linear demand function case. If firm 1 assesses a higher $E[c]$, then the intercept and slope of its reaction curve will change.

Examining the Q_1-intercept and the Q_2-intercept for firm 1, it follows that

$$\frac{\partial Q_2\text{-intercept}}{\partial E[\tilde{c}]} = \frac{1}{b} < 0$$

and

$$\frac{\partial Q_1\text{-intercept}}{\partial E[\tilde{c}]} = \frac{1}{2b} < 0.$$

Thus, as firm 1 assesses a higher mean, its reaction curve shifts downward to the left. Refer to figure 10.

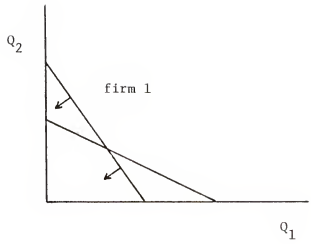

Figure 10

It can be seen that as this shift occurs, the equilibrium output changes. Q_2 becomes larger and Q_1 decreases. It may happen that Q_1 becomes zero and firm 2 takes over the entire market. Thus, if the assessment of the mean of firm 1 is higher than that of firm 2, firm 1 will produce less than firm 2. The same result occurs for all previous assumptions about the utility functions of the firms.

Summary

The introduction of cost uncertainty into the Cournot model leads to results significantly different from the original model. If one or both of the duopolists are risk averse, industry output declines. This result holds for any downward sloping demand curve. Once the equilibrium output is reached, it remains stable. That is, there is no incentive to move to a different output level.

An examination of the comparative statics of the model shows

that an increase in the mean or the variance of the density function of cost will lead to decreased output. Similarly, if one firm assesses a higher mean, that firm will produce less. Risk aversion guarantees this result.

CHAPTER VI
CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The results and conclusions evolving from this study are summarized in the following paragraphs. A study of the literature indicated that, in recent years, an extensive investigation has been made of the theory of the firm under uncertainty. The uncertainty, however, has been principally restricted to the demand function. The research that has focused on the specific area of input price uncertainty is limited and the result of this research has been inconclusive or ambiguous. The duopolist's behavior in a stochastic cost environment has not been previously studied.

The analysis of the competitive firm facing uncertain factor costs resulted in significantly different optimal allocations strategies than those of the neoclassical model. First, the expected utility maximizer may not choose capital and labor such that it is operating at its cost minimizing capital-labor combination. The entrepreneur is influenced by his degree of risk aversion. Second, it was shown that there may be circumstances under which the risk-averse firm may prefer uncertainty to certainty. Finally, the comparative statics of the stochastic model were determined and the results indicate that the entrepreneur, whose utility function displays decreasing absolute risk aversion, reacts to changes in the

distribution of random input prices as the firm under certainty would react to a change in the factor costs. These results reflect the growing opinion that individual firm behavior should be analyzed in a stochastic environment.

The analysis of the competitive firm was based on the assumption that one factor price is random while the other is fixed. This assumption reflects the fact that many firms contract for their supply of labor. Uncertainty plays a significant role in negotiations for these contracts. The risk attitudes of the bargaining parties influence the range of acceptable wages. It was shown that the ability to accurately assess the probability distribution of the wage rate and the utility function of the opposing party is essential for a bargaining range to be defined. Uncertainty increases the range of acceptable wages, thus, creating an advantage to the more powerful bargainer.

The introduction of cost uncertainty into the Cournot model resulted in the derivation of optimal behavior that differed from the original model. First, risk aversion causes the output of the duopolists to decline, but, as long as the firms are equally risk averse, a stable equilibrium will still exist. Second, differing risk attitudes may lead to the exit of one firm from the market. These results hold for any downward sloping demand function. Third, risk aversion causes the firms to decrease their outputs as the mean or variance of the probability density function increases. Finally, it was shown that differing cost expectations may result in changes

in output. That is, the firm assessing a higher mean will produce less.

Recommendations

In the course of carrying out this study, several potentially useful areas of research were revealed. The immediate extension of this research would be to consider the effects of input price uncertainty on the monopolistic firm.

Another extension would be to analyze the sources of the randomness in costs. That is, the models could be modified to include a component that directly represents the uncertainty in the factor supply functions.

This study focused on cost uncertainty. It would be worthwhile to consider cost uncertainty in conjunction with demand or production function uncertainty.

For further investigation, it is suggested that a bargaining model be developed that would incorporate uncertainty. Bayesian game theory may be an appropriate method for analyzing bargaining behavior under uncertainty.

While this dissertation examined the Cournot model under cost uncertainty, it would be of interest to analyze other duopoly models in an environment of uncertainty. Finally, it is suggested that by assuming specific functional forms for the random variable, it would be possible to examine firm behavior from a Bayesian point of view. That is, as the firm collects information regarding costs, it could update the model and revise its optimal behavior.

APPENDIX

APPENDIX A

$$
\begin{aligned}
(U'(\pi))^2 &= [U'(\bar{\pi}) + (\pi - \bar{\pi})U''(\bar{\pi}) \\
&\quad + (\pi - \bar{\pi})^2 \frac{U'''(\bar{\pi})}{2}]^2 \\
&= [U'(\bar{\pi})]^2 + (\pi - \bar{\pi})]^2 \\
&\quad + (\pi - \bar{\pi})^4 [\frac{U'''(\bar{\pi})}{2}]^2 \\
&\quad + 2U'(\bar{\pi})(\pi - \bar{\pi})U''(\bar{\pi}) \\
&\quad + 2U'(\bar{\pi})(\pi - \bar{\pi})^2 \frac{U'''(\bar{\pi})}{2} \\
&\quad + 2(\pi - \bar{\pi})^3 U''(\bar{\pi})\frac{U'''(\bar{\pi})}{2}.
\end{aligned}
$$

Taking the expected value of each side

$$
\begin{aligned}
E[(U'(\pi))^2] &= [U'(\bar{\pi})]^2 + \sigma_\pi^2 [U''(\bar{\pi})]^2 \\
&\quad + \sigma_\pi^2 \cdot \sigma_\pi^2 \cdot 3[\frac{U'''(\bar{\pi})}{2}]^2 \\
&\quad + U'(\bar{\pi})\sigma_\pi^2 \cdot U'''(\bar{\pi}) \\
&= [U'(\bar{\pi})]^2 + \sigma_\pi^2 [U''(\bar{\pi}))^2 \\
&\quad + U'(\bar{\pi})\ U'''(\bar{\pi})] \\
&\quad + 3\sigma_\pi^2 \sigma_\pi^2 [\frac{U'''(\bar{\pi})}{2}]^2.
\end{aligned}
$$

APPENDIX B

$$w^* - E[w] = \frac{\rho L\sigma_w^{\,2}[(U''(\bar{\pi}))^2 + 2L^2\sigma_w^{\,2}(\frac{U'''(\bar{\pi})}{2})^2]^{1/2}}{U'(\bar{\pi}) + L^2\sigma_w^{\,2}\,\frac{U'''(\bar{\pi})}{2}}.$$

Let

$$x = U''(\bar{\pi})^2 + 2L^2\sigma_w^{\,2}(\frac{U'''(\bar{\pi})}{2})^2.$$

Then,

$$\begin{aligned}
\frac{\partial w^*}{\partial \sigma_w} = \{ & \frac{\partial \rho}{\partial \sigma} L\sigma^2[x]^{1/2} \cdot U'(\bar{\pi}) \\
& + \frac{\partial \rho}{\partial \sigma} L^3\sigma^4[x]^{1/2} \cdot \frac{U'''(\bar{\pi})}{2} \\
& + \rho L\sigma[x]^{1/2} \cdot U'(\bar{\pi}) \\
& + 2\rho L^3\sigma^3[x]^{-1/2}(\frac{U'''(\bar{\pi})}{2})^2 \cdot U'(\bar{\pi}) \\
& + 2\rho L^5\sigma^5[x]^{-1/2} \cdot (\frac{U'''(\bar{\pi})}{2})^3 \\
& + \rho L\sigma[x]^{1/2} \cdot U'(\bar{\pi})\} \\
& \div [U'(\bar{\pi}) + L^2\sigma^2\,\frac{U'''(\bar{\pi})}{2}]^2 \\
= \{ & \sigma\frac{\partial \rho}{\partial \sigma}(L^3\sigma^3 U''(\bar{\pi})^2\,\frac{U'''(\bar{\pi})}{2}) \\
& + (\sigma\frac{\partial \rho}{\partial \sigma} + \rho)(2L^5\sigma^5(\frac{U'''(\bar{\pi})}{2})^3)
\end{aligned}$$

$$+ \left(\sigma\frac{\partial\rho}{\partial\sigma} + 2\rho\right)\left(L\sigma U'(\bar{\pi})(U''(\bar{\pi}))^2\right)$$

$$+ \left(\sigma\frac{\partial\rho}{\partial\sigma} + 3\rho\right)\left(2L^3\sigma^3\left(\frac{U'''(\bar{\pi})}{2}\right)^2\right.$$

$$\left. \cdot\ U'(\bar{\pi})\right)\}$$

$$\div\ [x]^{1/2}[U'(\bar{\pi}) + L^2\sigma^2\ \frac{U'''(\bar{\pi})}{2}]^2.$$

Where

$$\sigma = \sigma_w.$$

BIBLIOGRAPHY

BIBLIOGRAPHY

Apostol, T. Calculus (Vol. 1-2). Waltham, Massachusetts: Blaisdell, 1964.

Arrow, K. J. Theory of risk-bearing. Chicago: Markham, 1971.

Baron, D. Price uncertainty, utility, and industry equilibrium in pure competition. International Economic Review, October 1970.

Batra, R., & Ullah, A. Competitive firm and the theory of input demand under price uncertainty. Journal of Political Economy, November 1974.

Blair, R. Random input prices and the theory of the firm. Economic Inquiry, June 1974.

Blair, R., & Heggestad, A. The impact of uncertainty upon the multiproduct firm. Southern Economic Journal, July 1977.

Dhrymes, P. J. On the theory of the monopolistic multiproduct firm under uncertainty. International Economic Review, September 1964.

Doeleman, J., & Stahl, C. Uncertainty, price inflation, and wage indexation. Australian Economic Papers, December 1975.

Hartman, R. Competitive firm and the theory of input demand under price uncertainty: Comment. Journal of Political Economy, November 1975.

Henderson, J., & Quandt, R. Microeconomic theory. New York: McGraw-Hill, 1971.

Holthausen, D. M. Input choices and uncertain demand. American Economic Review, March 1976.

Horowitz, I. The price-quoter under risk. Western Economic Journal, June 1969.

Horowitz, I. Nondogmatic conjectures in a Cournot market. Western Economic Journal, March 1970.

Hosomatsu, Y. A note on the stability conditions in Cournot's dynamic market solution when neither the actual market demand function nor the production levels of rivals are known. Review of Economic Studies, January 1969.

Hymans, S. H. The price-taker: Uncertainty, utility, and the supply function. International Economic Review, September 1966.

Leland, H. Theory of the firm facing uncertain demand. American Economic Review, June 1972.

McCall, J. Probabilistic microeconomics. Bell Journal of Economics and Management Science, Autumn 1971.

Mills, E. Uncertainty and price theory. Quarterly Journal of Economics, February 1959.

Nelson, R. Uncertainty, prediction, and competitive equilibrium. Quarterly Journal of Economics, February 1961.

Oi, W. The desirability of price instability under perfect competition. Econometrica, January 1961.

Penner, R. G. Uncertainty and the short-run shifting of the corporation tax. Oxford Economic Papers, March 1967.

Pratt, J. Risk aversion in the small and in the large. Econometrica, January , 1964.

Rothchild, M., & Stiglitz, J. E. Increasing risk: I. A definition. Journal of Economic Theory, April 1970.

Rothenberg, T., & Smith, K. The effect of uncertainty on resource allocation in a general equilibrium model. Quarterly Journal of Economics, August 1971.

Sandmo, A. On the theory of the competitive firm under price uncertainty. American Economic Review, March 1971.

Sato, R., & Nagatani, K. The stability of oligopoly wity conjectural variations. Review of Economic Studies, October 1967.

Tarr, D. Stability in a Cournot market characterized by uncertainty. Western Economic Journal, June 1972.

Tisdell, C. Uncertainty, instability, expected profit. Econometrica, January-April, 1963.

von Neumann, J., & Morgenstern, O. Theory of games and economic behavior. Princeton, New Jersey: Princeton University Press, 1944.

Zabel, E. Monopoly and uncertainty. Review of Economic Studies, April 1970.

Zucker, A. On the desirability of price instability: An extension of the discussion. Econometrica, April 1965.

BIOGRAPHICAL SKETCH

Richard George Flood was born in Ypsilanti, Michigan, on October 13, 1947. He attended grade school and high school in Hammond, Indiana. He received his B.S. in Business Administration from Indiana University in 1972. He completed the degree requirements for his Ph.D. in 1978 at the University of Florida. He is presently on the faculty at the College of William and Mary. His wife, Lynn, resides with him in Williamsburg, Virginia.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Ira Horowitz, Chairman
Professor of Management

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

H. Russell Fogler
Associate Professor of Management

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Roger D. Blair
Associate Professor of Economics

This dissertation was submitted to the Graduate Faculty of the Department of Management in the College of Business Administration and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August 1978

_____

Dean, Graduate School